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DETERMINATION OF COSMOLOGICAL PARAMETERS

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ABSTRACT. Rapid progress has been made recently toward the measurement of cosmological parameters. Still, there are areas remaining where future progress will be relatively slow and difficult, and where further attention is needed. In this review, the status of measurements of the matter density (Ω_m), the vacuum energy density or cosmological constant (Ω_Λ), the Hubble constant (H_0), and ages of the oldest measured objects (t_0) are summarized. Many recent, independent dynamical measurements are yielding a low value for the matter density ($\Omega_m \sim 0.3$). New evidence from type Ia supernovae suggests that Ω_Λ may be non-zero. Many recent Hubble constant measurements appear to be converging in the range of 65-75 km/sec/Mpc. Eliminating systematic errors lies at the heart of accurate measurements for all of these parameters; as a result, a wide range of cosmological parameter space is currently still open. Fortunately, the prospects for accurately measuring cosmological parameters continue to increase and there is good reason for optimism that success may shortly be forthcoming.

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1. INTRODUCTION AND BRIEF HISTORICAL OVERVIEW

The recent success in the measurement of cosmological parameters can be attributed to a number of factors: an abundance of new observations, new approaches, and developments in detector technology (with a corresponding increase in the precision of the data). Given these advances, it is tempting to conclude that we have now entered an era of precision cosmology. Of course, whether this is indeed the case depends completely on the extent to which systematic measurement errors have been minimized or eliminated. In this context, it is interesting to view the measurement of cosmological parameters from a historical perspective as described briefly in the next section below. The present review concentrates primarily on results of the past couple of years, following on from a review on a similar topic given at the Texas Symposium in December, 1996 ([Freedman 1997a](#)).

The cosmological parameters discussed in this review are the following: the matter density $\Omega_m = 8\pi G \rho_m / 3H_0^2$, the Hubble parameter $H = \dot{a} / a$, (where a is the scale factor and H_0 , the Hubble constant, is the value at the current epoch), the vacuum energy density $\omega\Lambda = \Lambda / 3H_0^2$, and the age of the universe, t_0 . In Big Bang cosmology, the Friedmann equation relates the density, geometry and evolution of the universe: $H^2 = 8\pi G \rho_m / 3 - k / a^2 + \Lambda / 3$, where the average mass density is specified by ρ_m . The curvature term is specified by $\Omega_k = -k / a_0^2 H_0^2$, and for the case of a flat universe ($k = 0$), $\Omega_m + \omega\Lambda = 1$. A lower limit to the age, t_0 , can be determined by dating the oldest objects in the Universe. Or, alternatively, given an independent knowledge of the other cosmological parameters (H_0 , ω_m , $\omega\Lambda$, and Ω_k), a dynamical age of the Universe can also be determined by integrating the Friedmann equation.

Before moving on to describe recent developments, it is interesting to view how the values for these parameters have changed over time. In [Figures 1a-b](#) and [2a-b](#), estimates of H_0 , ω_m , λ , and t_0 are shown as a function of time. The historical discussion below is not intended to be comprehensive, but rather to outline the general trends as shown in [Figures 1](#) and [2](#). Each of the figures is discussed in turn.

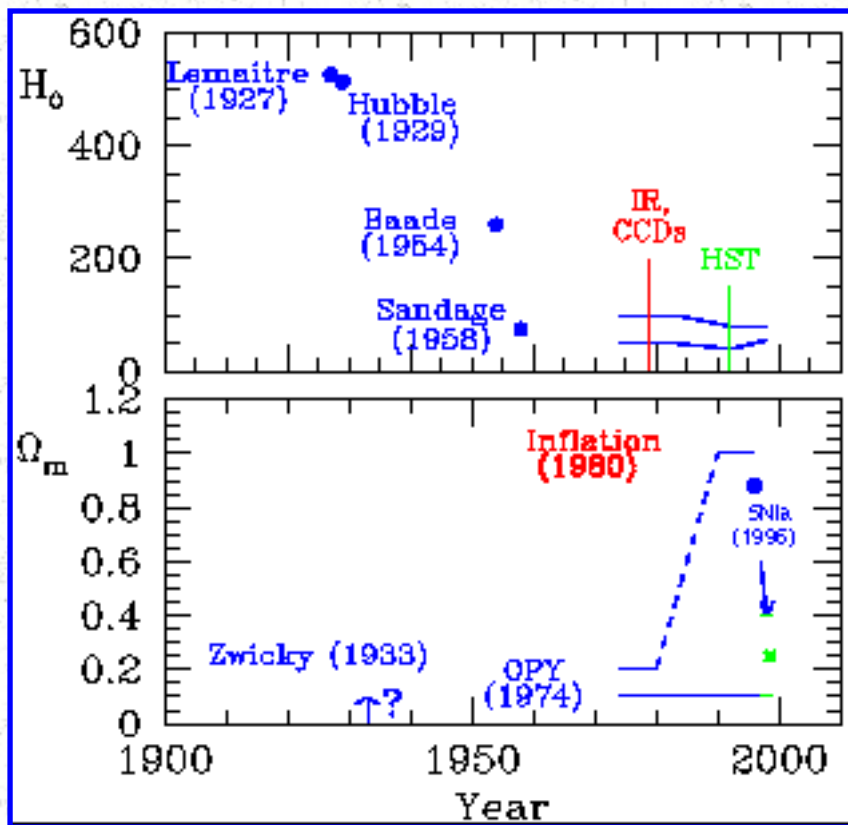


Figure 1a-b. The trend with time for measurements of H_0 and Ω_m . See text for details.

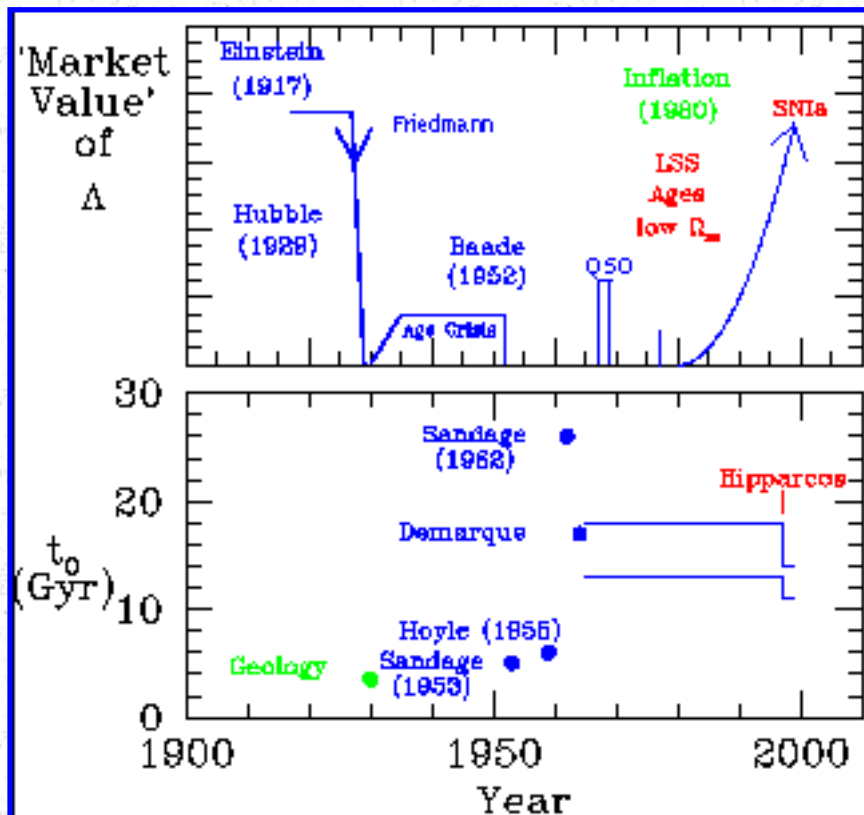


Figure 2a-b. The trend with time for Λ and t_0 . Note the arbitrary units for Λ . See text for details.

H_0 (Figure 1a): Over the last half century, the value of H_0 , the expansion rate of the Universe, has remained a well-known source of disagreement (for historical reviews see, for example, [van den Bergh 1997](#); [Rowan-Robinson 1985](#)). After [Baade \(1954\)](#) recalibrated the Cepheid distance scale, and [Sandage \(1958\)](#) recognized that the brightest stars in galaxies were ionized HII regions, the Hubble constant decreased from its original value of over 500, and fell into the well-known range of a factor of two, loosely constrained, as shown by the schematic lines drawn in [Figure 1a](#), between about 50 and 100 km/sec/Mpc. As indicated in the figure and discussed further below, recent improvements have come about as a result of new instrumentation and the availability of the [Hubble Space Telescope \(HST\)](#); in addition, several new methods for measuring distances to galaxies have been developed (see also [Livio, Donahue & Panagia 1997](#); [Freedman 1997b](#)). Recently, there has been some convergence in the value of H_0 . Although decreasing, the dominant errors in H_0 remain systematic in nature.

Ω_m (Figure 1b): [Zwicky \(1933\)](#) provided evidence that there was possibly 10-100 times more mass in the [Coma cluster](#) than contributed by the luminous matter in galaxies. However, it was not until the 1970's that the existence of dark matter began to be taken more seriously. At that time, evidence began to mount that showed rotation curves did not fall off with radius (e.g., [Rogstad & Shostak 1972](#); [Rubin et al. 1978](#); and [Bosma 1981](#)) and that the dynamical mass was increasing with scale from that of individual galaxies up through groups of galaxies (e.g., [Ostriker, Peebles & Yahil 1974](#)). A comprehensive historical review of the dark matter issue can be found in [Trimble \(1987\)](#). By the 1970's, the evidence was consistent with a total matter density of ~ 10 -20% of a critical density, ($\Omega = 1$) universe. With the development of inflation in the early 1980's ([Guth 1981](#)), tremendous effort was aimed at discovering both the nature and the amount of dark matter. In the early 1990's (see, for example, the review by [Dekel, Burstein & White 1997](#)), a number of studies indicated that we live in a critical density universe, and the first results for high redshift supernovae ([Perlmutter 1997](#)) were also consistent with a high matter density. As described below, however, the new supernovae results, and a wide range of other studies are consistent (once again) with a lower matter density of $\Omega_m \sim 0.2$ to 0.3. The overall trends in Ω_m with time are shown in [Figure 1b](#). The solid and dotted lines represent approximate upper and lower bounds only.

Λ (Figure 2a): The reader is referred to excellent overviews of the cosmological constant by [Weinberg \(1989\)](#) and [Carroll, Press & Turner \(1992\)](#). Enthusiasm for a non-zero value of Λ has come and gone several times over this century. For fun, I have plotted (in arbitrary units), the "market value" for shares of Λ in [Figure 2a](#). Here it can be seen that the market for Λ has been quite volatile over time. Shares for Λ were high when [Einstein \(1917\)](#) first introduced this cosmological constant in an attempt to allow for a stable universe; subsequent work, for example, by [Friedmann \(1922\)](#), followed by the discovery of the expansion of the Universe by Edwin Hubble, led to the crash of Λ (along with the rest of the stock market) in 1929!

The value of H_0 measured by [Hubble \(1929\)](#) implied a dynamical age for the Universe of only ~ 2 billion years. This age was younger than the geological dating estimates for the age of the Earth, placed then at about 3.5 billion years. This discrepancy led to an "age crisis", and a renewal of interest in Λ , that was eventually solved by Baade's recalibration of the distance scale in 1954. A brief period of activity occurred in the Λ market with the observation by [Petrosian et al. \(1967\)](#), of an apparent peak at $z \sim 2$ in the quasar distribution. However, as more quasars were observed, this feature also disappeared.

In general, consumers have tended to be wary of stock in Λ due to the difficulties of explaining the current limits in conflict by 120 orders of magnitude with the predictions of the standard model of particle physics (e.g., [Weinberg 1989](#)). However, recently, as [Figure 2a](#) shows, the low observed matter density, the recurring conflict in ages between some values of H_0 and globular cluster ages, and the observed large-scale distribution of galaxies have motivated a renewed interest in Λ (e.g., [Krauss & Turner 1995](#); [Ostriker & Steinhardt 1995](#)). Just as for other commodities, the consumer may need to be concerned about how inflation is driving the market. It is probably too early to say if the bull market of the 1990's is over - this is an area where, as in the world of the stock market, the experts disagree!

t_0 ([Figure 2b](#)): In the 1950's, the first applications of stellar evolution models to determine ages for globular clusters resulted in ages somewhat older than the age of the Earth; these estimates climbed considerably to about 26 billion years in the early 1960's as illustrated in [Figure 2b](#). Much of this increase resulted from a change in the adopted helium abundance (see the historical review by [Demarque et al. 1991](#)). As described by Demarque et al. (and references therein), the value of 5 billion years obtained by Sandage in 1953 assumed a helium abundance of $Y = 0.4$, whereas the value of 26 billion years obtained by Sandage in 1962 was based on an adopted value of $Y = 0.1$. The age estimates began to stabilize once it was recognized that the helium abundance for globular clusters was closer to that of the Sun ($Y \sim 0.25$). The ages of globular clusters have remained at approximately 15-16 billion years (bracketed loosely by the bounds shown schematically in the figure) for some time; only recently, with the new results from the Hipparcos satellite (plus new opacities) have the ages again dropped systematically. These new results are discussed further below.

These cartoons illustrate a couple of simple and obvious points. Ultimately, values of cosmological parameters will not be determined by market value or popular enthusiasm; they must come from accurate experiments. But as also indicated in these plots, such measurements are difficult, they are dominated by systematic uncertainties, and so require a very high level of testing, independent measurements and scrutiny, before we can know with confidence if convergence (if any) is real.

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2. DETERMINATION OF Ω_m

On the scale sizes of clusters of galaxies $\sim 1 h^{-1}$ Mpc, many techniques have been applied to estimate Ω_m (e.g., Bahcall, this conference; [Bahcall & Fan 1998](#); [Dekel, Burstein & White 1997](#)). The bottom line is that the apparent matter density appears to amount to only ~ 20 -30% of the critical density required for a flat, $\Omega = 1$ universe. In fact, the most recent data are consistent with most of the extant data from the past 20 years. Cluster velocity dispersions ([Carlberg et al. 1996](#)), the distortion of background galaxies behind clusters or weak lensing ([Kaiser & Squires 1993](#); [Smail et al. 1995](#)), the baryon density in clusters ([White et al. 1993](#)), and the existence of very massive clusters at high redshift ([Bahcall & Fan](#)), all currently favor a low value of the matter density ($\Omega_m \sim 0.2$ -0.3), at least on scales up to about $2h^{-1}$ Mpc.

However, measurements at scales larger than that of clusters are extremely challenging, and determinations of Ω_m have not yet converged (e.g., see [Dekel, Burstein & White 1997](#)). For example, measurements of peculiar velocities of galaxies, have led independent groups to come to very different conclusions, with estimates of Ω_m ranging from about 0.2 to 1.3. Dekel, Burstein & White conclude that the peculiar velocity results yield $\Omega_m > 0.3$ at the $2\text{-}\sigma$ level. A new weak lensing study of a supercluster ([Kaiser et al. 1999](#)) on a scale of $6 h^{-1}$ Mpc, yields a (surprisingly) low value of Ω_m (~ 0.05), under the assumption that there is no bias in the way that mass traces light. Small & Sargent (1998) have recently probed the matter density for the [Corona Borealis supercluster](#) (at a scale of $\sim 20 h^{-1}$ Mpc), finding $\Omega_m \sim 0.4$. Under the assumption of a flat universe, global limits can also be placed on Ω_m from studies of type Ia supernovae (see next section); currently the supernova results favor a value $\Omega_m \sim 0.3$.

The measurement of the total matter density of the Universe remains an important and challenging problem. It should be emphasized that all of the methods for measuring Ω_m are based on a number of underlying assumptions. For different methods, the list includes diverse assumptions about how the mass distribution traces the observed light distribution, whether clusters are representative of the Universe, the properties and effects of dust grains, or the evolution of the objects under study. The accuracy of any matter density estimate must ultimately be evaluated in the context of the validity of the underlying assumptions upon which the method is based. Hence, it is non-trivial to assign a quantitative uncertainty in many cases but, in fact, systematic effects (choices and assumptions) may be the dominant source of uncertainty.

An exciting result has emerged this year from atmospheric neutrino experiments undertaken at Superkamiokande (Totsuka, this volume), providing evidence for vacuum oscillations between muon and another neutrino species, and a lower limit to the mass in neutrinos. The contribution of neutrinos to the total density is likely to be small, although interestingly it may be comparable to that in stars.

Determining whether there is a significant, smooth underlying component to the matter density on the largest scales is a critical issue that must be definitively resolved. If, for example, some or all of the non-baryonic dark matter is composed of very weakly interacting particles, that component could prove very elusive and difficult to detect. It unfortunately remains the case that at present, it is not yet possible to distinguish *unambiguously and definitively* among $\Omega_m = 1$, $\Omega_m + \Omega_\Lambda = 1$, and open universes with $\Omega_0 < 1$, models all implying very different underlying fundamental physics. The preponderance of evidence at the present time, however, does not favor the simplest case of $\Omega_m = 1$ (the Einstein-de Sitter universe).

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3. DETERMINATION OF Ω_Λ

As illustrated in [Figure 2a](#), the cosmological constant Λ has had a long and volatile history in cosmology. There have been many reasons to be skeptical about a non-zero value of the cosmological constant. To begin with, there is a discrepancy of ≥ 120 orders of magnitude between current observational limits and estimates of the vacuum energy density based on current standard particle theory (e.g. [Carroll, Press and Turner 1992](#)). A further difficulty with a non-zero value for Λ is that it appears coincidental that we are now living at a special epoch when the cosmological constant has begun to affect the dynamics of the Universe (other than during a time of inflation). It is also difficult to ignore the fact that historically a non-zero Λ has been called upon to explain a number of other apparent crises, and moreover, adding additional free parameters to a problem always makes it easier to fit data.

However, despite the strong arguments have been made for $\Lambda = 0$, there are growing reasons for a renewed interest in a non-zero value. Although the current value of Λ is small compared to the observed limits, there is no known physical principle that demands $\Lambda = 0$ (e.g., [Carroll, Press & Turner 1992](#)). Although Einstein originally introduced an arbitrary constant term, standard particle theory and inflation now provide a physical interpretation of Λ : it is the energy density of the vacuum (e.g., [Weinberg 1989](#)). Finally, a number of observational results can be explained with a low Ω_m and $\Omega_m + \Omega_\Lambda = 1$: for instance, the observed large scale distribution of galaxies, clusters, and voids described previously, in addition to the recent results from type Ia supernovae described below. In addition, the discrepancy between the ages of the oldest stars and the expansion age (exacerbated if $\Omega_m = 1$) can be resolved.

Excitement has recently been generated by the results from two groups studying type Ia supernovae at high redshift (one team's results were reported at this meeting by Ariel Goobar). Both groups have found that the high redshift supernovae are fainter (and therefore further), on average, than implied by either an open ($\Omega_m = 0.2$) or a flat, matter-dominated ($\Omega_m = 1$) universe. The observed differences are ~ 0.25 and 0.15 mag, ([Reiss et al. 1998](#) and [Perlmutter et al. 1998a](#), respectively), or equivalently $\sim 13\%$ and 8% in distance. A number of tests have been applied to search for possible systematic errors that might produce this observed effect, but none has been identified. Taken at face value, these results imply that the vacuum energy density of the Universe, (Λ), is non-zero.

The early results from these two groups have evolved as more data have become available. [Perlmutter et al. \(1997\)](#) first reported results based on a sample of 7 high-redshift ($z \sim 0.4$) supernovae. Initially, they found evidence for a high matter density $\omega_m \sim 0.9 \pm 0.3$, with a value of Ω_Λ consistent with zero. However, with the subsequent discovery of a $z \sim 0.8$ supernova, [Perlmutter et al. \(1998a\)](#) found instead that a low-mass density ($\Omega_m \sim 0.2$) universe was preferred. The second, independent group obtained preliminary results based on 4 supernovae which were also consistent with a lower matter density ([Garnavich et al. 1998](#)). The sample sizes have now grown larger, with 10 supernovae being reported by

[Reiss et al. \(1998\)](#) and 42 supernovae being reported by [Perlmutter et al. \(1999\)](#). These two new larger data sets are yielding consistent conclusions, and the supernovae are now indicating a non-zero and positive value for $\Omega_\Lambda \sim 0.7$, and a small matter density, $\Omega_m \sim 0.3$, under the assumption that $\Omega_m + \Omega_\Lambda = 1$. If a flat universe is not assumed, the best fit to the Perlmutter et al. data yields $\Omega_m = 0.73$, $\Omega_\Lambda = 1.32$. The Hubble diagram for both the nearby (Hamuy et al. 1996) and the distant ([Reiss et al. 1998](#)) samples of supernovae are shown in [Figure 3](#).

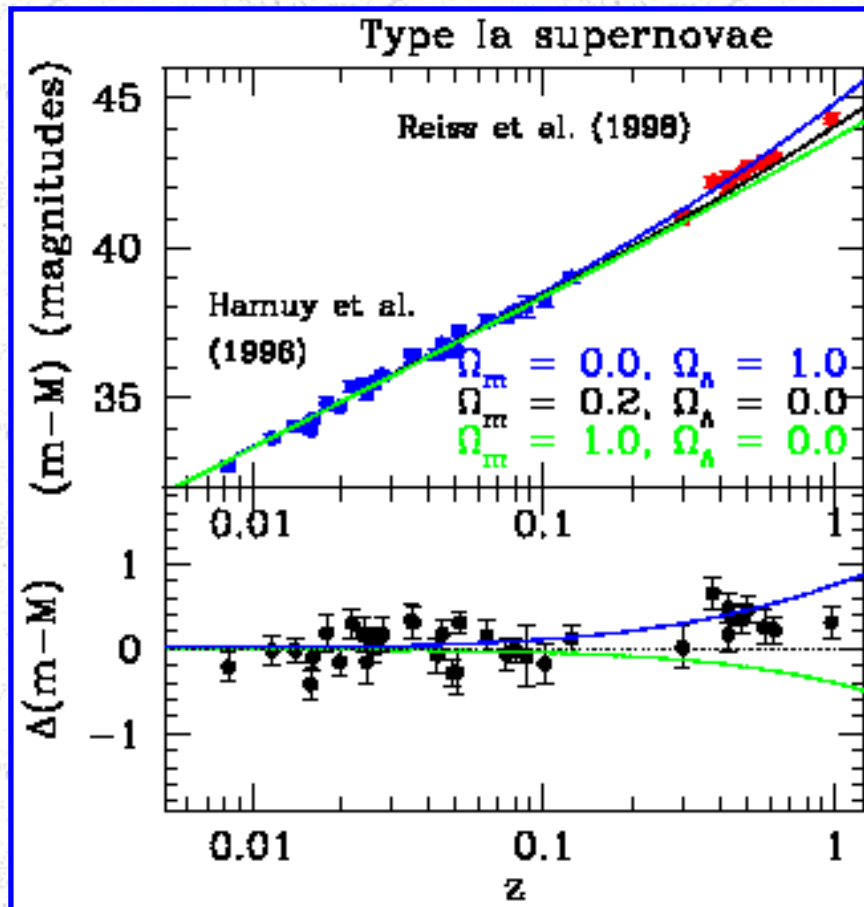


Figure 3 (top panel): The Hubble diagram for type Ia supernovae from Hamuy et al. (1996) and [Reiss et al. \(1998\)](#). Plotted is the distance modulus in magnitudes versus the logarithm of the redshift. Curves for various cosmological models are indicated. **(bottom panel):** Following [Reiss et al. \(1998\)](#), the difference in magnitude between the observed data points compared to an open ($\Omega_m = 0.2$) model is shown. The distant supernovae are fainter by 0.25 magnitudes, on average, than the nearby supernovae.

The advantages of using type Ia supernovae for measurements of Ω_Λ are many. The dispersion in the nearby type Ia supernova Hubble diagram is very small (0.12 mag or 6% in distance, as reported by [Reiss et al. 1996](#)). They are bright and therefore can be observed to large distances. In principle, at $z \sim 1$, the

shape of the Hubble diagram alone can be used to separate Ω_m and $\omega\Lambda$, independent of the nearby, local calibration sample (Goobar & Perlmutter 1995). Potential effects due to evolution, chemical composition dependence, changing dust properties are all amenable to empirical tests and calibration.

A possible weakness of all of the current supernova $\omega\Lambda$ studies is that the luminosities of the high-redshift supernovae are all measured relative to the same set of local supernovae. Although in the future, estimates of $\Omega\Lambda$ at high redshift will be possible using the shape of the Hubble diagram alone (Goobar & Perlmutter 1995), at present, the evidence for $\omega\Lambda$ comes from a differential comparison of the nearby sample of supernovae at $z < 1$, with those at $z \sim 0.3-0.8$. Hence, the absolute calibrations, completeness levels, and any other systematic effects pertaining to both datasets are critical. For several reasons, the search techniques and calibrations of the nearby and the distant samples are different. Moreover, the intense efforts to search for high-redshift objects have now led to the situation where the nearby sample is now smaller than the distant samples. While the different search strategies may not necessarily introduce systematic differences, increasing the nearby sample will provide an important check. Such searches are now underway by several groups.

Although a 0.25 mag difference between the nearby and distant samples appears large, the history of measurements of H_0 provides an interesting context for comparison. In the case of H_0 determinations, a difference of 0.25 mag in zero point only corresponds to a difference between 60 and 67 km/sec/Mpc! Current differences in the published values for H_0 result from a number of arcane factors: the adoption of different calibrator galaxies, the adoption of different techniques for measuring distances, treatment of reddening and metallicity, and differences in adopted photometric zero point. In fact, despite the considerable progress on the extragalactic distance scale and the Hubble constant, recent H_0 values tend to range from about 60 to 80 km/sec/Mpc (see next section). (As recently as five years ago, there was a factor of 2 discrepancy in these values, corresponding to a difference of 1.5 mag.)

In interpreting the observed difference between nearby and distant supernovae, it is also important to keep in mind that, for the known properties of dust in the interstellar medium, the ratio of total-to-selective absorption, ($R_B = A_B / E(B - V)$), (the value by which the colors are multiplied to correct the blue magnitudes), is ~ 4 . Hence, very accurate photometry and colors are required to ultimately understand this issue. A relative error of only 0.03 mag in color could contribute 0.12 mag to the observed difference in magnitude.

Further tests and limits on Λ may come from gravitational lens number density statistics (Fukugita et al. 1990; Fukugita and Turner 1991; Kochanek 1996), plus more stringent limits to the numbers of close-separation lenses. The numbers of strong gravitational lenses detected depends on the volume surveyed; hence, the probability that a quasar will be lensed is a very sensitive function of $\Omega\Lambda$. In a flat universe with $\Omega\Lambda = 0$, almost an order of magnitude fewer lenses are predicted than for a universe with $\Omega\Lambda = 1$.

If the current results from supernovae are correct, then the numbers of close-separation lenses should be significantly larger than predicted for $\Lambda = 0$ models. Complications for the lens number density statistics

arise due to a number of factors which are hard to quantify in an error estimate, and which become increasingly more important for smaller values of Λ : for example, galaxies evolve (and perhaps merge) with time, galaxies contain dust, the properties of the lensing galaxies are not well-known (in particular, the dark matter velocity dispersion is unknown), and the numbers of lensing systems for which this type of analysis has been carried out is still very small. However, the sample of known lens systems is steadily growing, and new limits from this method will be forthcoming.

The gravitational lens number density limits from Kochanek (1996) are $\Omega\lambda < 0.66$ (95% confidence) for $\Omega_m + \Omega\lambda = 1$. However, more recently, [Cheng & Krauss \(1998\)](#) have reinvestigated the sensitivity of this method to various factors. As Kochanek and Cheng & Krauss have underscored, the uncertainties in modelling of the lensing galaxies (generally as isothermal spheres with core radii), the observed luminosity functions, core radii of the galaxies, and the resulting magnification bias (that results due to the fact that the lensed quasar images are amplified, and hence, easier to detect than if there were no lensing) all need to be treated carefully. Also, as Cheng & Krauss emphasize, the optical depth for lensing depends on the velocity dispersion to the fourth power and hence, better accuracies in the velocity dispersions are required. Cheng & Krauss conclude that systematic uncertainties currently dominate the results from this method, but that a flat universe with a low value of Ω_m cannot be excluded.

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4. DETERMINATION OF H_0

The range of both previous and current published values for the expansion rate, or Hubble constant, H_0 (see [Figure 1a](#)), attest to the difficulty of measuring this parameter accurately. Fortunately, the past 15 years has seen a series of substantive improvements leading toward the measurement of a more accurate value of H_0 . Indeed, it is quite likely that the 1- σ uncertainty in H_0 is now approaching 10%, a significant advance over the factor-of-two uncertainty that lingered for decades. Briefly, the significant progress can be mainly attributed to the replacement of photographic cameras (used in this context from the 1920's to the 1980's) by solid-state detectors, as well as to both the development of several completely new, and the refinement of existing, methods for measuring extragalactic distances and H_0 (e.g., Livio, Donahue & Panagia 1997; [Freedman 1997b](#)).

Currently there are many empirical routes to the determination of H_0 ; these fall into the following completely independent and very broad categories: 1) the gravitational lens time delay method, 2) the Sunyaev-Zel'dovich method for clusters, and 3) the extragalactic distance scale. In the latter category, there are several independent methods for measuring distances on the largest scales (including supernovae), but most of these methods share common, empirical calibrations at their base. In the future, another independent determination of H_0 , from measurements of anisotropies in the cosmic microwave background, may also be feasible, if the physical basis for the anisotropies can be well-established.

Each of the above methods carries its own susceptibility to systematic errors, but the methods as listed here, have completely independent systematics. If history in this field has taught us nothing else, it offers the following important message: *systematic errors have dominated, and continue to dominate, the measurement of H_0* . It is therefore vital to measure H_0 using a variety of methods, and to test for the systematics that are affecting each of the different kinds of techniques.

Not all of these methods have yet been tested to the same degree. Important progress is being made on all fronts; however, some methods are still limited by sample size and small-number statistics. For example, method 1), the gravitational time delay method, has only two well-studied lens systems to date: 0957+561 and PG 1115. The great advantage of both methods 1) and 2), however, is that they measure H_0 at very large distances, independent of the need for any local calibration.

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4.1 Gravitational Lenses

Refsdael ([1964](#), [1966](#)) noted that the arrival times for the light from two gravitationally lensed images of a background point source are dependent on the path lengths and the gravitational potential traversed in each case. Hence, a measurement of the time delay and the angular separation for different images of a variable quasar can be used to provide a measurement of H_0 . This method offers tremendous potential because it can be applied at great distances and it is based on very solid physical principles ([Blandford & Kundic 1997](#)).

There are of course difficulties with this method as there are with any other. Astronomical lenses are galaxies whose underlying (luminous or dark) mass distributions are not independently known, and furthermore they may be sitting in more complicated group or cluster potentials. A degeneracy exists between the mass distribution of the lens and the value of H_0 (e.g., Keeton and Kochanek 1997; [Schechter et al. 1997](#)). Ideally velocity dispersion measurements as a function of position are needed (to constrain the mass distribution of the lens). Such measurements are very difficult (and generally have not been available). Perhaps worse yet, the distribution of the dark matter in these systems is unknown.

Unfortunately, to date, there are very few systems known which have both a favorable geometry (for providing constraints on the lens mass distribution) and a variable background source (so that a time delay can be measured). The two systems to date that have been well-studied yield values of H_0 in the approximate range of 40-70 km/sec/Mpc ([Schechter et al. 1997](#); [Impey et al. 1998](#)) with an uncertainty of ~ 20 -30%. These values assume a value of $\omega = 1$, and rise by 10% for low Ω . [Tonry & Franx \(1998\)](#) have recently reported an accurate new velocity dispersion of $\sigma = 288 \pm 9$ km/sec for the lensing galaxy in 0957+561, based on data obtained at the Keck 10m telescope. Adopting $\omega_m = 0.25$ and the model of [Grogan & Narayan \(1996\)](#) for the mass distribution of the lens yields a value of $H_0 = 72 \pm 7$ (1- σ statistical) $\pm 15\%$ (systematic).

As the number of favorable lens systems increases (as further lenses are discovered that have measurable time delays), the prospects for measuring H_0 and its uncertainty using this technique are excellent. Schechter (private communication) reports that there are now 6 lenses with measured time delays, but perhaps only half of these will be useful for H_0 determinations due to the difficulty of modelling the galaxies.

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4.2 Sunyaev Zel'dovich Effect and X-Ray Measurements

The inverse-Compton scattering of photons from the cosmic microwave background off of hot electrons in the X-ray gas of rich clusters results in a measurable decrement in the microwave background spectrum known as the Sunyaev-Zel'dovich (SZ) effect ([Sunyaev & Zel'dovich 1969](#)). Given a spatial distribution of the SZ effect and a high-resolution X-ray map, the density and temperature distributions of the hot gas can be obtained; the mean electron temperature can be obtained from an X-ray spectrum. The method makes use of the fact that the X-ray flux is distance-dependent, whereas the Sunyaev-Zel'dovich decrement in the temperature is not.

Once again, the advantages of this method are that it can be applied at large distances and, in principle, it has a straightforward physical basis. Some of the main uncertainties result from potential clumpiness of the gas (which would result in reducing H_0), projection effects (if the clusters observed are prolate, H_0 could be larger), the assumption of hydrostatic equilibrium, details of the models for the gas and electron densities, and potential contamination from point sources.

To date, a range of values of H_0 have been published based on this method ranging from ~ 40 -80 km/sec/Mpc (e.g., [Birkinshaw 1998](#)). The uncertainties are still large, but as more and more clusters are observed, higher-resolution (2D) maps of the decrement, and X-ray maps and spectra become available, the prospects for this method are improving enormously.

Carlstrom, (this meeting) presented exquisite new measurements of the Sunyaev-Zel'dovich decrement, measured in two dimensions with an interferometer, for a number of nearby clusters. He also highlighted the imminent advances on the horizon for X-ray imaging using [NASA's](#) soon-to-be launched Chandra X-ray Observatory (the satellite formerly known as AXAF). There is promise of a significant increase in accuracy for this method in the near future.

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4.3 The Extragalactic Distance Scale

The launch of the [Hubble Space Telescope \(HST\)](#) in 1990 has provided the opportunity to undertake a major program to calibrate the extragalactic distance scale. The resolution of [HST](#) is an order of magnitude higher than can be generally obtained through the Earth's atmosphere, and moreover it is stable; as a result, the volume of space made accessible by [HST](#) increased by 3 orders of magnitude. The [HST](#) Key Project on the extragalactic distance scale was designed to measure the Hubble constant to an accuracy of $\pm 10\%$ *rms* (Freedman et al. 1994; Mould et al. 1995; [Kennicutt et al. 1995](#)). Since the dominant sources of error are systematic in nature, the approach we have taken in the Key Project is to measure H_0 by intercomparing several different methods. This approach allows us to assess and quantify explicitly the systematic errors. The [HST](#) Key Project will be completed in 1999; since new results will be available shortly, this discussion will be kept very brief. Results based on half of the available data yield $H_0 = 72 \pm 5 \pm (\text{random}) 7 (\text{systematic})$ km/sec/Mpc ([Madore et al. 1998, 1999](#); [Freedman et al. 1998](#); [Mould et al. 1997](#)). In [Figure 4](#), the results for various H_0 methods are combined using both a Frequentist and a Bayesian approach (from Madore et al.).

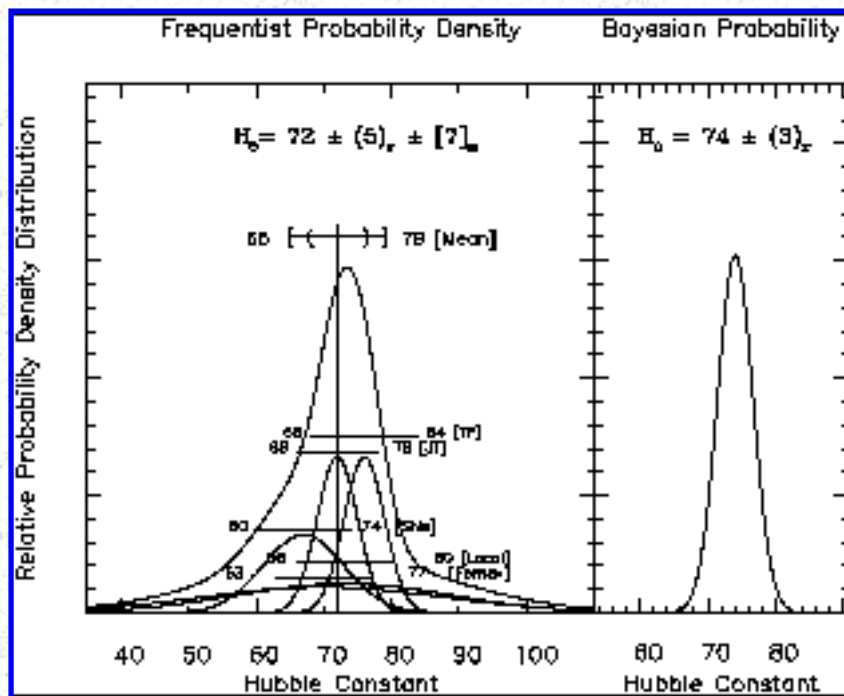


Figure 4. Plot of various H_0 determinations and the adopted values from [Madore et al. \(1998\)](#). In the left panel, each value of H_0 and its statistical uncertainty is represented by a Gaussian of unit area (linked dotted line) centered on its determined value and having a dispersion equal to the quoted random error. Superposed immediately above each Gaussian is a horizontal bar representing the one sigma limits of the calculated systematic errors derived for that determination. The adopted average value and its probability distribution function (continuous solid line) is the arithmetic sum of the individual Gaussians. This Frequentist representation treats each determination as independent, and assumes no *a priori* reason to prefer one solution over another. A Bayesian representation of the products of the various probability density distributions is shown in the right panel. Because of the close proximity and strong overlap in the various independent solutions the Bayesian estimator is very similar to, while more sharply defined than, the Frequentist solution.

The largest remaining sources of uncertainty in the extragalactic distance route to H_0 can be traced to uncertainty in the distance to the [Large Magellanic Cloud](#) (the galaxy which provides the fiducial comparison for more distant galaxies), and to the potential effects of differing amounts of elements heavier than helium (or metallicity). The importance of the latter effect has been difficult to establish. The recently-installed infrared (NICMOS) camera on [HST](#) is being used to address this, and may help to resolve the issue shortly.

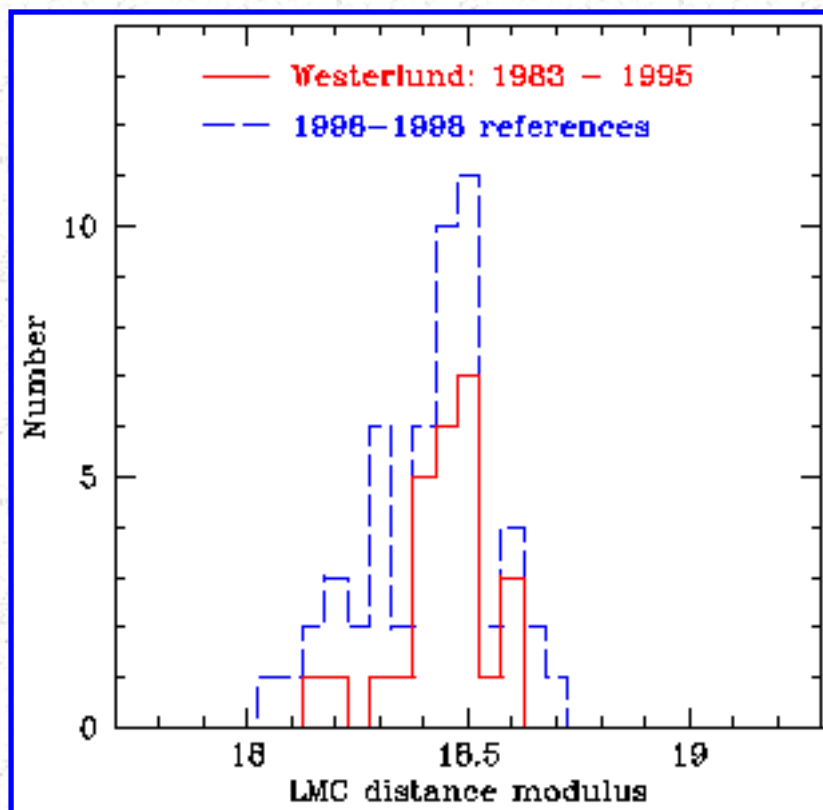


Figure 5. A histogram of distance moduli determinations for the [Large Magellanic cloud](#). Values prior to 1996 are from a published compilation by [Westerlund \(1997\)](#).

A histogram of the distribution of distances to the [LMC](#) from the literature is shown in [Figure 5](#). The distances in this histogram are based on Cepheids, RR Lyraes, [SN 1987A](#), red giants, the "red clump", and long-period variables. Values prior to 1996 come from the published compilation of [Westerlund \(1997\)](#), but only the latest revision published by a given author is plotted for a given data set. Despite decades of effort in measuring the distance to this nearby neighboring galaxy, and the number of independent methods available, the dispersion in measured distance modulus remains very high. Moreover, the distribution is not Gaussian. There has been much recent activity on the red clump which contributes many of the values around 18.3 mag, and gives rise to the bimodal nature of the distribution. There is as yet no understanding of why there is a systematic difference between the Cepheid and the red clump distance scale. This histogram illustrates that the uncertainty in the distance to the [LMC](#) is still large. Without assuming that the distribution is Gaussian, the 95% confidence limits are ± 0.28 mag, and the 68% confidence limits amount to ± 0.13 mag or 7% in distance. Unfortunately, the distance to the [LMC](#) remains as one of the largest systematic uncertainties in the current extragalactic distance scale.

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5. DETERMINATION OF t_0

Age-dating of the oldest known objects in the Universe has been carried out in a number of ways. The most reliable ages are generally believed to come from the application of theoretical models of stellar evolution to observations of the oldest clusters in the Milky Way, the globular clusters. For about 30 years, the ages of globular clusters have remained reasonably stable, at about 15 billion years (e.g., [Demarque et al. 1991](#); [Vandenbergh et al. 1996](#)); however, recently these ages have been revised downward, as described below. Ages can also be obtained from radioactive dating or nucleocosmochronology (e.g., [Schramm 1989](#)), and a further lower limit can be estimated from the cooling rates for white dwarfs (e.g., [Oswalt et al. 1996](#)). Generally, these ages have ranged from about 10 to 20 billion years; the largest sources of uncertainty in each of these estimates are again systematic in nature.

During the 1980's and 1990's, the globular cluster age estimates have improved as both new observations of globular clusters have been made with CCD's, and as refinements to stellar evolution models, including updated opacities, consideration of mixing, and different chemical abundances, have been incorporated (e.g., [Vandenbergh et al. 1996](#); [Chaboyer et al. 1996, 1998](#)). The latter authors have undertaken an extensive analysis of the uncertainties in the determination of globular cluster ages. From the theory side, uncertainties in globular cluster ages result, for example, from uncertainties in how convection is treated, the opacities, and nuclear reaction rates. From the measurement side uncertainties arise due to corrections for dust and chemical composition; however, *the dominant source of overall uncertainty in the globular cluster ages is the uncertainty in the cluster distances.*

In fact, the impact of distance uncertainty on the ages of globular clusters is twice as large as its effect on the determination of H_0 . That is, a 0.2 mag difference in zero point corresponds to a 10% difference in the distance (or correspondingly, H_0), but it results in a 20% difference in the age of a cluster (e.g., [Renzini 1991](#)).

The Hipparcos satellite has recently provided geometric parallax measurements for 118,000 nearby stars ([Kovalevsky 1998](#)). Relevant for the calibration of globular cluster distances, are the relatively nearby old stars of low metal composition, the so-called subdwarf stars, presumably the nearby analogs of the old, metal-poor stars in globular clusters. Accurate distances to these stars provide a fiducial calibration from which the absolute luminosities of equivalent stars in globular clusters can be determined and compared with those from stellar evolution models. The new Hipparcos calibration has led to a downward revision of the globular cluster ages from ~ 15 billion years to 11-14 billion years (e.g., [Reid 1997](#); [Pont et al. 1998](#); [Chaboyer et al. 1998](#)).

However, as emphasized by Chaboyer et al., there are only 8 stars in the Hipparcos catalog having both

small parallax errors, and low metal abundance, $[\text{Fe}/\text{H}] < -1.1$ (i.e., less than one tenth the iron-to-hydrogen abundance relative to the solar value). In fact, Gratton et al. (1998) note that there are no stars with parallax errors $< 10\%$ with $[\text{Fe}/\text{H}] < -2$ corresponding to the oldest, metal poor globular clusters. Hence, the calibration of globular cluster ages based on parallax measurements of old, metal-poor stars remains an area where an increase in sample size will be critical to beat down the statistical uncertainties. A decade or so from now, new parallax satellites such as [NASA's SIM](#) (the Space Interferometry Mission) and the European Space Agency's mission (named GAIA) will be instrumental in improving these calibrations, not only for subdwarfs, but for many other classes of stars (for example, Cepheids and the lower-mass variable RR Lyrae stars). These interferometers will be capable of delivering 2-3 orders of magnitude more accurate parallaxes than Hipparcos, down to fainter magnitude limits for several orders of magnitude more stars. Until larger samples of accurate parallaxes are available, however, distance errors are likely to continue to contribute the largest source of systematic uncertainty to the globular cluster ages.

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6. THE COSMIC MICROWAVE BACKGROUND RADIATION AND COSMOLOGICAL PARAMETERS

As discussed at this meeting by Silk, Wilkinson and Spergel, over the next few years, increasingly more accurate measurements will be made of the fluctuations in the cosmic microwave background (CMB) radiation. The underlying physics governing the shape of the CMB anisotropy spectrum can be described by the interaction of a very tightly coupled fluid composed of electrons and photons before recombination (e.g., [Hu & White 1996](#); [Sunyaev & Zel'dovich \(1970\)](#). [Figure 6](#) shows a plot of the predicted angular power spectrum for CMB anisotropies from Hu, Sugiyama & Silk (1997), computed under the assumption that the fluctuations are Gaussian and adiabatic. The position of the first angular peak is very sensitive to ω_0 ($\Omega_m + \omega\Lambda + \Omega_k$).

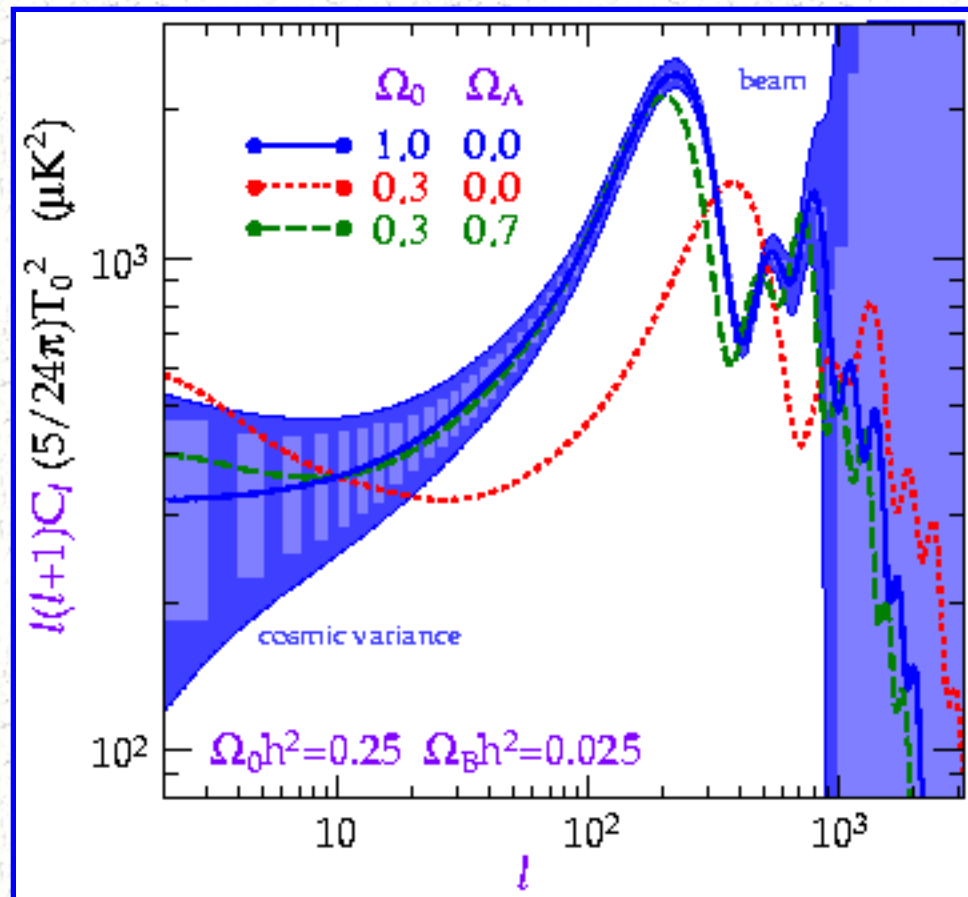


Figure 6. The angular power spectrum of cosmic microwave background anisotropies assuming adiabatic, nearly scale-invariant models for a range of values of ω_0 and Ω_Λ (Hu, Sugiyama, and Silk 1997; their Figure 4). The C_l values correspond to the squares of the spherical harmonics coefficients. Low l values correspond to large angular scales ($l \sim 200^\circ / \theta$). The position of the first acoustic peak is predicted to be at $l \sim 220 \Omega_{\text{TOT}}^{-1/2}$, and hence, shifts to smaller angular scales for open universes.

For information on cosmological parameters to be extracted from the CMB anisotropies, the following must be true: first, the physical source of these fluctuations must be understood, and second, the sources of systematic error must be eliminated or minimized so that they do not dominate the uncertainties.

Recently it has become clear that almost exact degeneracies exist between various cosmological parameters (e.g., [Efstathiou & Bond 1998](#); [Eisenstein, Hu & Tegmark 1998](#)) such that, for example, cosmological models with the same matter density can have the same CMB anisotropies, while having very different geometries. As a result, measurement of CMB anisotropies will, in principle, be able to yield strong constraints on the products $\Omega_m h^2$ and $\Omega_b h^2$, but not on the individual values of h ($= H_0 / 100$) and Ω_m directly. Hence, earlier suggestions that such cosmological parameters could be measured from CMB anisotropies to precisions of 1% or better (e.g., Bond, Efstathiou & Tegmark 1997) will unfortunately not be realized. However, breaking these degeneracies can be accomplished by using the CMB data in combination with other data, for example, the Sloan survey and type Ia supernovae (e.g. [White 1999](#)).

Currently the estimates of the precisions for which cosmological parameters can be extracted from measurements of anisotropies in the CMB are based on models in which the primordial fluctuations are Gaussian and adiabatic, and for which there is no preferred scale. Very detailed predictions can be made for this model, more so than for competing models such as isocurvature baryons or cosmic strings or textures. In the next few years, as the data improve, all of these models will be scrutinized in greater detail.

Important additional constraints may eventually come from polarization measurements (e.g., [Zaldarriaga, Spergel & Seljak 1997](#); [Kamionkowski et al. 1997](#)), but these may require the next generation of experiments beyond MAP and Planck. The polarization data may provide a means of breaking some of the degeneracies amongst the cosmological parameters that are present in the temperature data alone. Furthermore, they are sensitive to the presence of a tensor (gravity wave) contribution, and hence can allow a very sensitive test of competing models.

Although it is not yet certain how accurately the cosmological parameters can be extracted from measurements of CMB anisotropies, what is clear is that upcoming, scheduled balloon and space

experiments offer an opportunity to probe detailed physics of the early Universe. If current models are correct, the first acoustic peak will be confirmed very shortly and its position accurately measured by balloon experiments even before the launch of MAP. These balloon experiments will soon be followed with the total sky and multi-frequency coverage provided by MAP and Planck. This new era now being entered, of precision CMB anisotropy experiments, is extremely exciting.

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7. DISCUSSION AND SUMMARY

In the past year, a radical shift has begun to occur. Until recently, a majority of the theoretical community viewed the (standard) Einstein-de Sitter model ($\Omega_0 = 1$, $\Omega\Lambda = 0$) as the most likely case (with $h = 0.5$, $t = 13$ Gyr). With accumulating evidence for a low matter density, difficulty in fitting the galaxy power spectrum with the standard model, the conflict in ages for the Einstein-de Sitter case, and now, most recently, the evidence from type Ia supernovae for an accelerating universe, a new "standard" model is emerging, a model with $\Omega_m \sim 0.3$, $\Omega\Lambda \sim 0.7$, $h = 0.65$, and $t = 13$ Gyr. This model preserves a flat universe and is still consistent with inflation.

In [Figure 7](#), the bounds on several cosmological parameters are summarized in a plot of the matter density as a function of the Hubble constant. What do these current limits on cosmological parameters imply about the contribution of non-baryonic dark matter to the total matter density? As can be seen from the figure, for $H_0 = 70$ km/sec/Mpc, current limits to the deuterium abundance ([Burles & Tytler 1998](#); [Hogan 1998](#)) yield baryon densities in the range of $\Omega_b = 0.02$ to 0.04 , or 2-4% of the critical density. Given current estimates of the matter density ($\Omega_m \sim 0.3$), non-baryonic matter would thus contribute just over 25% of the total energy density needed for a flat, $\Omega = 1$ universe.

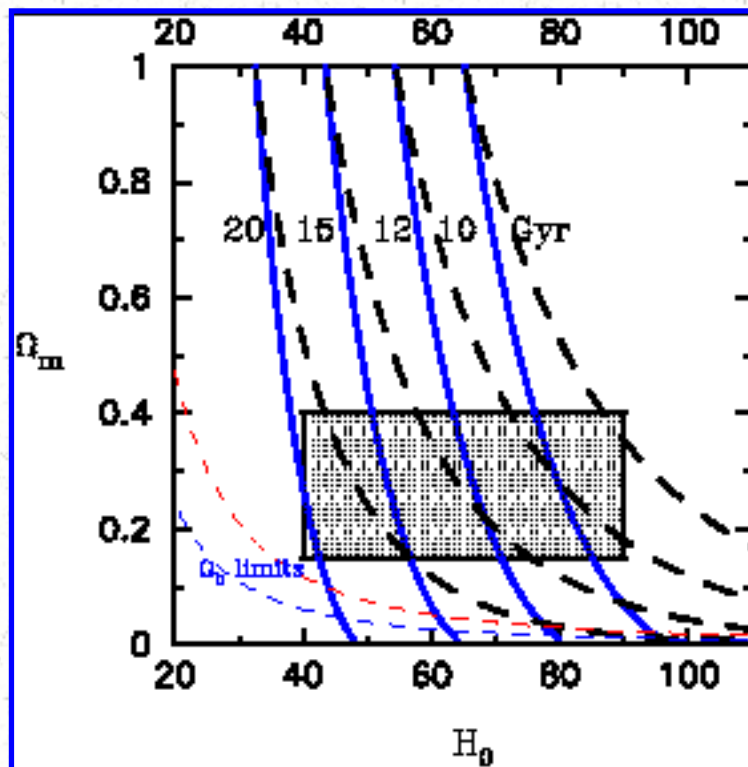


Figure 7. Plot of ω_m versus H_0 showing current observational limits on cosmological parameters. The shaded box is defined by values of H_0 in the range of 40 to 90 km/sec/Mpc and $0.15 < \Omega_m < 0.4$. The thick solid lines denote expansion ages for an open ($\Omega_\Lambda = 0$) Universe for 10, 15, and 20 Gyr and the thick dashed lines denote expansion ages in the case of a flat ($\Omega_m + \Omega_\Lambda = 1$) Universe. The light dashed lines denote current limits for ω_b based on low and high values for the deuterium to-hydrogen ratio.

One might ask, is non-baryonic dark matter still required if Λ is non-zero? Allowance for $\Lambda \neq 0$ does not provide the missing energy to simultaneously yield $\Omega = 1$, while doing away with the necessity of non-baryonic dark matter, at least for the current limits from Big Bang nucleosynthesis. As can be seen from [Figure 7](#), for the current deuterium limits, having all baryonic mass plus Λ would require both $H_0 \sim 30$ km/sec/Mpc and an age for the Universe of ~ 30 Gyr. These H_0 and age values are outside the range of currently measured values for both of these parameters. Although it might be appealing to do away simultaneously with one type of unknown (non-baryonic dark matter) while introducing another parameter (Λ), a non-zero value for the cosmological parameter does not remove the requirement for non-baryonic dark matter.

The question of the nature of dark matter (or energy) remains with us. In this sense, the situation has not changed very much over the past few decades, although the motivation for requiring a critical-density universe has evolved from considerations of fine-tuning and causal arguments to the development of inflation. But searches for dark matter since the 1970's have not uncovered sufficient matter to result in a critical-density universe. This year has offered exciting new (and therefore still tentative) results that a non-zero value of the cosmological constant, or perhaps an evolving scalar field like quintessence (Steinhardt, this volume; [Steinhardt & Caldwell 1998](#)) could provide the missing energy to result in a critical-density universe. Still, the nature of the dark matter, whether it contributes 25% or 95% of the total energy density, is unknown, and remains as one of the most fundamental unsolved problems in cosmology.

The progress in measuring cosmological parameters has been impressive; still, however, the accurate measurement of cosmological parameters remains a challenging task. It is therefore encouraging to note the wealth of new data that will appear over the next few years, covering very diverse areas of parameter space. For example, measurement of CMB anisotropies, (from balloons and space with MAP, and Planck), the [Sloan Digital Sky Survey](#), [Hubble Space Telescope](#), Chandra X-ray Observatory, radio interferometry, gravitational lensing studies, weakly interacting massive particle (WIMP) cryogenic detectors, neutrino experiments, the [Large Hadron Collider \(LHC\)](#), and a host of others, inspire optimism that the noose on cosmological parameters is indeed tightening. At the very least, the next few years should continue to be interesting!

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