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COSMOLOGICAL PARAMETERS

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Abstract. The cosmological parameters that I will emphasize are the Hubble parameter $H_0 \equiv 100h$ km s^{-1} Mpc^{-1}, the age of the universe t_0, the average matter density Ω_m, the baryonic matter density Ω_b, the neutrino density $\Omega\nu$, and the cosmological constant $\Omega\Lambda$. The evidence currently favors $t_0 \approx$ 13 Gyr, $h \approx 0.65$, $\Omega_m \approx 0.4 \pm 0.1$, $\Omega_b = 0.02h^{-2}$, $0.001 < \Omega\nu < 0.1$, and $\Omega\Lambda \approx 0.7$.

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1. INTRODUCTION

In this review I will concentrate on the values of the cosmological parameters. The other key questions in cosmology today concern the nature of the dark matter and dark energy, the origin and nature of the primordial inhomogeneities, and the formation and evolution of galaxies. I have been telling my theoretical cosmology students for several years that these latter topics are their main subjects for research, since determining the values of the cosmological parameters is now mainly in the hands of the observers.

In discussing cosmological parameters, it will be useful to distinguish between two sets of assumptions: (a) general relativity plus the assumption that the universe is homogeneous and isotropic on large scales (Friedmann-Robertson-Walker framework), or (b) the FRW framework plus the Λ CDM family of models. In addition to the FRW framework, the Λ CDM models assume that the present matter density Ω_m plus the cosmological constant (or its equivalent in ‘‘dark energy’’) in units of critical density $\Omega\Lambda = \Lambda / (3 H_0^2)$ sum to unity ($\Omega_m + \Omega\Lambda = 1$) to produce the flat universe

predicted by simple cosmic inflation models. These ΛCDM models assume that the primordial fluctuations were adiabatic (all components fluctuate together) and Gaussian, and had a Zel'dovich spectrum ($P_p(k) = A\,k^n$, with $n \approx 1$), and that the dark matter is mostly of the cold variety.

Although the results from the Long-Duration BOOMERANG [30, 75] and the MAXIMA-1 [54, 5] CMB observations and analyses [59] were were not yet available at the Dark Matter 2000 conference, I have made use of them in preparing this review. The table below summarizes the current observational information about the cosmological parameters, with estimated 1σ errors. The quantities in brackets have been deduced using at least some of the ΛCDM assumptions. The rest of this paper discusses these issues in more detail. But it should already be apparent that there is impressive agreement between the values of the parameters determined by various methods.

Table 1. Cosmological Parameters [results assuming CDM in brackets]

H_0	$= 100\,h$ km s^{-1} Mpc^{-1} , $h = 0.65 \pm 0.08$
t_0	= 9-16 Gyr (from globular clusters)
	= [9-17 Gyr from expansion age, ΛCDM models]
Ω_b	$= (0.045 \pm 0.0057)\,h_{65}^{-2}$ (from D/H)
	$>$ [$0.04\,h_{65}^{-2}$ from Lyα forest opacity]
Ω_m	$= 0.4 \pm 0.2$ (from cluster baryons)
	= [0.34 ± 0.1 from Lyα forest $P(k)$]
	= [0.4 ± 0.2 from cluster evolution]
	> 0.3 (2.4σ , from cosmic flows)
	= [0.5 ± 0.1 from flows plus SN Ia]
	$\approx 3/4\,\Omega\Lambda - 1/4 \pm 1/8$ from SN Ia
$\Omega_m + \Omega\Lambda$	$= 1.11 \pm 0.07$ (from CMB peak location)
$\Omega\Lambda$	$= 0.71 \pm 0.14$ (from previous two lines)
	$<$ & 0.73 (2σ) from radio QSO lensing
$\Omega\nu$	$\gtrsim 0.001$ (from Superkamiokande)
	$\lesssim$ [0.1]

2. AGE OF THE UNIVERSE t_0

The strongest lower limits for t_0 come from studies of the stellar populations of globular clusters (GCs). In the mid-1990s the best estimates of the ages of the oldest GCs from main sequence turnoff magnitudes were $t_{GC} \approx 15$ - 16 Gyr [15, 124, 23]. A frequently quoted lower limit on the age of GCs was 12 Gyr [23], which was then an even more conservative lower limit on $t_0 = t_{GC} + \Delta t_{GC}$, where

$\langle t_{GC} \rangle$ ~ 0.5 Gyr is the time from the Big Bang until GC formation. The main uncertainty in the GC age estimates came from the uncertain distance to the GCs: a 0.25 magnitude error in the distance modulus translates to a 22% error in the derived cluster age [22].

In spring of 1997, analyses of data from the Hipparcos astrometric satellite indicated that the distances to GCs assumed in obtaining the ages just discussed were systematically underestimated [101, 51]. It follows that their stars at the main sequence turnoff are brighter and therefore younger. Stellar evolution calculation improvements also lowered the GC age estimates. In light of the new Hipparcos data, Chaboyer et al. [24] have done a revised Monte Carlo analysis of the effects of varying various uncertain parameters, and obtained t_{GC} = 11.5±1.3 Gyr (1σ), with a 95% C.L. lower limit of 9.5 Gyr. The latest detailed analysis [20] gives t_{GC} = 11.5±2.6 Gyr from main sequence fitting using parallaxes of local subdwarfs, the method used in [101, 51]. These authors get somewhat smaller GC distances when all the available data is used, with a resulting t_{GC} = 12.9±2.9 Gyr (95% C.L.). However, if main sequence fitting is the more reliable method, the younger age may be more appropriate.

Stellar age estimates are of course based on stellar evolution calculations, which have also improved significantly. But the solar neutrino problem reminds us that we are not really sure that we understand how even our nearest star operates; and the sun plays an important role in calibrating stellar evolution, since it is the only star whose age we know independently (from radioactive dating of early solar system material). An important check on stellar ages can come from observations of white dwarfs in globular and open clusters [102].

What if the GC age estimates are wrong for some unknown reason? The only other non-cosmological estimates of the age of the universe come from nuclear cosmochronometry - radioactive decay and chemical evolution of the Galaxy - and white dwarf cooling. Cosmochronometry age estimates are sensitive to a number of uncertain issues such as the formation history of the disk and its stars, and possible actinide destruction in stars [79, 82]. However, an independent cosmochronometry age estimate of 15.6±4.6 Gyr has been obtained based on data from two low-metallicity stars, using the measured radioactive depletion of thorium (whose half-life is 14.2 Gyr) compared to stable heavy r-process elements [27, 28]. This method could become very important if it were possible to obtain accurate measurements of r-process element abundances for a number of very low metallicity stars giving consistent age estimates, and especially if the large errors could be reduced.

Independent age estimates come from the cooling of white dwarfs in the neighborhood of the sun. The key observation is that there is a lower limit to the luminosity, and therefore also the temperature, of nearby white dwarfs; although dimmer ones could have been seen, none have been found (cf. however [53]). The only plausible explanation is that the white dwarfs have not had sufficient time to cool to lower temperatures, which initially led to an estimate of 9.3±2 Gyr for the age of the Galactic disk [130]. Since there was evidence, based on the pre-Hipparcos GC distances, that the stellar disk of our Galaxy is about 2 Gyr younger than the oldest GCs (e.g., [121, 108]), this in turn gave an estimate of the age of the universe of $t_0 \approx$ 11±2 Gyr. Other analyses [132, 56] conclude that sensitivity to disk star formation history, and to effects on the white dwarf cooling rates due to C/O separation at crystallization and possible presence of trace elements such as ^{22}Ne, allow a rather wide range of ages for the disk of about 10±4 Gyr. One determination of the white dwarf luminosity function, using white dwarfs in proper motion binaries, leads to a somewhat lower minimum luminosity and therefore a somewhat higher estimate of the age of the disk of ~ $10.5^{+2.5}_{-1.5}$ Gyr [88]. More recent observations [76] and analyses [9] lead to an estimated age of the galactic disk of 8±1.5 Gyr.

We conclude that $t_0 \gtrsim 13$ Gyr, with ~ 10 Gyr a likely lower limit. Note that $t_0 > 13$ Gyr implies that $h \leq 0.50$ for matter density $\Omega_m = 1$, and that $h \leq 0.73$ even for Ω_m as small as 0.3 in flat cosmologies (i.e., with $\Omega_m + \Omega\Lambda = 1$).

3. HUBBLE PARAMETER H_0

The Hubble parameter $H_0 \equiv 100h$ km s^{-1} Mpc^{-1} remains uncertain, although no longer by the traditional factor of two. The range of h determinations has been shrinking with time [64]. De Vaucouleurs long contended that $h \gtrsim 1$. Sandage has long contended that $h \gtrsim 0.5$, although a recent reanalysis of the Type Ia supernovae (SNe Ia) data coauthored by Sandage and Tammann [111] concludes that the latest data are consistent with $h = 0.6 \pm 0.04$.

The Hubble parameter has been measured in two basic ways: (1) Measuring the distance to some nearby galaxies, typically by measuring the periods and luminosities of Cepheid variables in them; and then using these ''calibrator galaxies'' to set the zero point in any of the several methods of measuring the relative distances to galaxies. (2) Using fundamental physics to measure the distance to some distant object(s) directly, thereby avoiding at least some of the uncertainties of the cosmic distance ladder [109]. The difficulty with method (1) was that there was only a handful of calibrator galaxies close enough for Cepheids to be resolved in them. However, the HST Key Project on the Extragalactic Distance Scale has significantly increased the set of calibrator galaxies. The difficulty with method (2) is that in every case studied so far, some aspect of the observed system or the underlying physics remains somewhat uncertain. It is nevertheless remarkable that the results of several different methods of type (2) are rather similar, and indeed not very far from those of method (1). This gives reason to hope for convergence.

3.1. Relative Distance Methods

One piece of good news is that the several methods of measuring the relative distances to galaxies now mostly seem to be consistent with each other. These methods use either ''standard candles'' or empirical relations between two measurable properties of a galaxy, one distance-independent and the other distance-dependent. The favorite standard candle is SNe Ia, and observers are now in good agreement. Taking account of an empirical relationship between the SNe Ia light curve shape and maximum luminosity leads to $h = 0.65 \pm 0.06$ [103], $h = 0.64^{+0.08}_{-0.06}$ [61], or $h = 0.63 \pm 0.03$ [52, 93], and the slightly lower value mentioned above from the latest analysis coauthored by Sandage and Tammann agrees within the errors. The HST Key Project result using SNe Ia is $h = 0.65 \pm 0.02 \pm 0.05$, where the first error quoted is statistical and the second is systematic [50], and their Cepheid metallicity-dependent luminosity-period relationship [65] has been used (this lowers h by 4%). Some of the other relative distance methods are based on old stellar populations: the tip of the red giant branch (TRGB), the planetary nebula luminosity function (PNLF), the globular cluster luminosity function (GCLF), and the surface brightness fluctuation method (SBF). The HST Key Project result using these old star standard candles is [43] $h = 0.66 \pm 0.04 \pm 0.06$, including the Cepheid metallicity correction. The old favorite empirical relation used as a relative distance indicator is the Tully-Fisher relation between the rotation velocity and luminosity of spiral galaxies. The ''final'' value of the Hubble constant from the HST Key Project taking all of these into account, including the metallicity dependence of the Cepheid period-luminosity relation, is [45] $h = 0.74 \pm 0.04 \pm 0.07$, where the first error is statistical and the second is systematic. The largest source of systematic uncertainty is the distance to the LMC, which is here assumed to have a distance modulus of 18.45. This is a significantly higher h than their previous [85] $h = 0.71 \pm 0.06$, or $h = 0.68 \pm 0.06$ including the Cepheid metallicity dependence, using a LMC distance modulus of 18.5.

3.2. Fundamental Physics Approaches

The fundamental physics approaches involve either Type Ia or Type II supernovae, the Sunyaev-Zel'dovich (S-Z) effect, or gravitational lensing of quasars. All are promising, but in each case the relevant physics remains somewhat uncertain.

The ^{56}Ni radioactivity method for determining H_0 using Type Ia SNe avoids the uncertainties of the distance ladder by calculating the absolute luminosity of Type Ia supernovae from first principles using plausible but as yet unproved physical models for ^{56}Ni production. The first result obtained was that $h = 0.61\pm0.10$ [3, 17]; however, another study [77] (cf. [126]) found that uncertainties in extinction (i.e., light absorption) toward each supernova increases the range of allowed h. Demanding that the ^{56}Ni radioactivity method agree with an expanding photosphere approach leads to $h = 0.60^{+0.14}_{-0.11}$ [86]. The expanding photosphere method compares the expansion rate of the SN envelope measured by redshift with its size increase inferred from its temperature and magnitude. This approach was first applied to Type II SNe; the 1992 result $h = 0.6\pm0.1$ [114] was subsequently revised upward by the same authors to $h = 0.73\pm0.06\pm0.07$ [115]. However, there are various complications with the physics of the expanding envelope [110, 35].

The S-Z effect is the Compton scattering of microwave background photons from the hot electrons in a foreground galaxy cluster. This can be used to measure H_0 since properties of the cluster gas measured via the S-Z effect and from X-ray observations have different dependences on H_0. The result from the first cluster for which sufficiently detailed data was available, A665 (at $z = 0.182$), was $h = (0.4 - 0.5)\pm0.12$ [13]; combining this with data on A2218 ($z = 0.171$) raised this somewhat to $h = 0.55\pm0.17$ [12]. The history and more recent data have been reviewed by Birkinshaw [14], who concludes that the available data give a Hubble parameter $h \approx 0.6$ with a scatter of about 0.2. But since the available measurements are not independent, it does not follow that $h = 0.6\pm0.1$; for example, there is a selection effect that biases low the h determined this way.

Several quasars have been observed to have multiple images separated by $\theta \sim$ a few arc seconds; this phenomenon is interpreted as arising from gravitational lensing of the source quasar by a galaxy along the line of sight (first suggested by [100]; reviewed in [129]). In the first such system discovered, QSO 0957+561 ($z = 1.41$), the time delay Δt between arrival at the earth of variations in the quasar's luminosity in the two images has been measured to be, e.g., 409±23 days [89], although other authors found a value of 540±12 days [94]. The shorter Δt has now been confirmed [72, 117]. Since $\Delta t \approx \theta^2 H_0^{-1}$, this observation allows an estimate of the Hubble parameter. The latest results for h from 0957+561, using all available data, are $h = 0.64\pm0.13$ (95% C.L.) [72], and $h = 0.62\pm0.07$ [39], where the error does not include systematic errors in the assumed form of the lensing mass distribution.

The first quadruple-image quasar system discovered was PG1115+080. Using a recent series of observations [113], the time delay between images B and C has been determined to be about 24±3 days. A simple model for the lensing galaxy and the nearby galaxies then leads to $h = 0.42\pm0.06$ [113], although higher values for h are obtained by more sophisticated analyses: $h = 0.60\pm0.17$ [63], $h = 0.52\pm0.14$ [73]. The results depend on how the lensing galaxy and those in the compact group of which it is a part are modelled.

Another quadruple-lens system, B1606+656, leads to $h = 0.59\pm0.08\pm0.15$, where the first error is the 95% C.L. statistical error, and the second is the estimated systematic uncertainty [41]. Time delays have also recently been determined for the Einstein ring system B0218+357, giving $h = 0.69^{+0.13}_{-0.19}$ (95% C.L.) [11].

Mainly because of the systematic uncertainties in modelling the mass distribution in the lensing systems, the uncertainty in the h determination by gravitational lens time delays remains rather large. But it is reassuring that this completely independent method gives results consistent with the other determinations.

3.3. Conclusions on H_0

To summarize, relative distance methods favor a value $h \approx 0.6$ - 0.8. Meanwhile the fundamental physics methods typically lead to $h \approx 0.4$ - 0.7. Among fundamental physics approaches, there has been important recent progress in measuring h via the Sunyev-Zel'dovich effect and time delays between different images of gravitationally lensed quasars, although the uncertainties remain larger than via relative distance methods. For the rest of this review, we will adopt a value of $h = 0.65 \pm 0.08$. This corresponds to $t_0 = 6.52h^{-1}$ *Gyr* = 10±2 Gyr for $\Omega_m = 1$ - probably too low compared to the ages of the oldest globular clusters. But for $\Omega_m = 0.2$ and $\Omega\Lambda = 0$, or alternatively for $\Omega_m = 0.4$ and $\Omega\Lambda = 0.6$, $t_0 = 13 \pm 2$ Gyr, in agreement with the globular cluster estimate of t_0. This is one of the weakest of the several arguments for low Ω_m, a non-zero cosmological constant, or both.

4. HOT DARK MATTER DENSITY $\Omega\nu$

The recent atmospheric neutrino data from Super-Kamiokande [46] provide strong evidence of neutrino oscillations and therefore of non-zero neutrino mass. These data imply a lower limit on the hot dark matter (i.e., light neutrino) contribution to the cosmological density $\Omega\nu \gtrsim 0.001$. $\Omega\nu$ is actually that low, and therefore cosmologically uninteresting, if $m(\nu\tau) >> m(\nu_\mu)$, as is suggested by the hierarchical pattern of the quark and charged lepton masses. But if the $\nu\tau$ and ν_μ are nearly degenerate in mass, as suggested by their strong mixing, then $\Omega\nu$ could be substantially larger. Although the Cold + Hot Dark Matter (CHDM) cosmological model with $h \approx 0.5$, $\Omega_m = 1$, and $\Omega\nu = 0.2$ predicts power spectra of cosmic density and CMB anisotropies that are in excellent agreement with the data [96, 49], as we have just seen the large value measured for the Hubble parameter makes such $\Omega_m = 1$ models dubious. It remains to be seen whether including a significant amount of hot dark matter in low-Ω_m models improves their agreement with data. Primack & Gross [97, 98] found that the possible improvement of the low-Ω_m flat (ΛCDM) cosmological models with the addition of light neutrinos appears to be rather limited, and the maximum amount of hot dark matter decreases with decreasing Ω_m [95]. For $\Omega_m \lesssim 0.4$, [29] find that $\Omega\nu \lesssim 0.08$; [47] finds more restrictive upper limits with the constraint that the primordial power spectrum index $n \leq 1$, but this may not be well motivated.

5. COSMOLOGICAL CONSTANT Λ

The strongest evidence for a positive Λ comes from high-redshift SNe Ia, and independently from a combination of observations indicating that $\Omega_m \sim 0.4$ together with CMB data indicating that the universe is nearly flat. We will discuss these observations in the next section. Here we will start by looking at other constraints on Λ.

The cosmological effects of a cosmological constant are not difficult to understand [42, 74, 21]. In the early universe, the density of energy and matter is far more important than the Λ term on the r.h.s. of the Friedmann equation. But the average matter density decreases as the universe expands, and at a rather low redshift ($z \sim 0.2$ for $\Omega_m = 0.3$, $\Omega\Lambda = 0.7$) the Λ term finally becomes dominant. Around this redshift, the Λ term almost balances the attraction of the matter, and the scale factor $a \equiv (1 + z)^{-1}$

increases very slowly, although it ultimately starts increasing exponentially as the universe starts inflating under the influence of the increasingly dominant Λ term. The existence of a period during which expansion slows while the clock runs explains why t_0 can be greater than for $\Lambda = 0$, but this also shows that there is an increased likelihood of finding galaxies in the redshift interval when the expansion slowed, and a correspondingly increased opportunity for lensing by these galaxies of quasars (which mostly lie at higher redshift $z \gtrsim 2$).

The observed frequency of such optical lensed quasars is about what would be expected in a standard $\Omega = 1$, $\Lambda = 0$ cosmology, so this data sets fairly stringent upper limits: $\Omega\Lambda \leq 0.70$ at 90% C.L. [81, 69], with more recent data giving even tighter constraints: $\Omega\Lambda < 0.66$ at 95% confidence if $\Omega_m + \Omega\Lambda = 1$ [70]. This limit could perhaps be weakened if there were (a) significant extinction by dust in the E/S0 galaxies responsible for the lensing or (b) rapid evolution of these galaxies, but there is much evidence that these galaxies have little dust and have evolved only passively for $z \lesssim 1$ [120, 78, 112]. An alternative analysis [58] of some of the same optical lensing data gives a value $\Omega\Lambda = 0.64_{-0.26}{}^{+0.15}$. My group [80] (cf. [7]) showed that edge-on disk galaxies can lens quasars very effectively, and discussed a case in which optical extinction is significant. But the radio observations discussed by [39], which give a 2σ limit $\Omega\Lambda < 0.73$, are not affected by extinction, so those are the ones quoted in the Table above. Recently a reanalysis [25] of lensing using new models of the evolution of elliptical galaxies gave $\Omega\Lambda = 0.7^{+0.1}{}_{-0.2}$, but Kochanek et al. [71] (see especially Fig. 4) show that the available evidence disfavors such models.

A model-dependent constraint appeared to come from simulations of Λ CDM [67] and OpenCDM [60] COBE-normalized models with $h = 0.7$, $\Omega_m = 0.3$, and either $\Omega\Lambda = 0.7$ or, for the open case, $\Omega\Lambda = 0$. These models have too much power on small scales to be consistent with observations, unless there is strong scale-dependent antibiasing of galaxies with respect to dark matter. However, recent high-resolution simulations [68] find that merging and destruction of galaxies in dense environments lead to exactly the sort of scale-dependent antibiasing needed for agreement with observations for the Λ CDM model. Similar results have been found using simulations plus semi-analytic methods [8] (but cf. [62]).

Another constraint on Λ from simulations is a claim [6] that the number of long arcs in clusters is in accord with observations for an open CDM model with $\Omega_m = 0.3$ but an order of magnitude too low in a Λ CDM model with the same Ω_m. This apparently occurs because clusters with dense cores form too late in such models. This is potentially a powerful constraint, and needs to be checked and understood. It is now known that including cluster galaxies does not alter these results [83, 44].

6. MEASURING Ω_m

The present author, like many theorists, has long regarded the Einstein-de Sitter ($\Omega_m = 1$, $\Lambda = 0$) cosmology as the most attractive one. For one thing, of the three possible constant values for Ω - 0, 1, and ∞ - the only one that can describe our universe is $\Omega_m = 1$. Also, cosmic inflation is the only known solution for several otherwise intractable problems, and all simple inflationary models predict that the universe is flat, i.e. that $\Omega_m + \Omega\Lambda = 1$. Since there is no known physical reason for a non-zero cosmological constant, it was often said that inflation favors $\Omega = 1$. Of course, theoretical prejudice is not a reliable guide. In recent years, many cosmologists have favored $\Omega_m \sim 0.3$, both because of the H_0 - t_0 constraints and because cluster and other relatively small-scale measurements have given low values for Ω_m. (For a summary of arguments favoring low ≈ 0.2 and $\Lambda = 0$, see [26]; [32] is a review that notes that larger scale measurements favor higher Ω_m.) But the most exciting new evidence has

come from cosmological-scale measurements.

Type Ia Supernovae. At present, the most promising techniques for measuring Ω_m and $\Omega\Lambda$ on cosmological scales use the small-angle anisotropies in the CMB radiation and high-redshift Type Ia supernovae (SNe Ia). We will discuss the latter first. SNe Ia are the brightest supernovae, and the spread in their intrinsic brightness appears to be relatively small. The Supernova Cosmology Project [90] demonstrated the feasibility of finding significant numbers of such supernovae. The first seven high redshift SNe Ia that they analyzed gave for a flat universe $\Omega_m = 1 - \Omega\Lambda = 0.94^{+0.34}_{-0.28}$, or equivalently $\Omega\Lambda = 0.06^{+0.28}_{-0.34}$ (< 0.51 at the 95% confidence level) [90]. But adding one $z = 0.83$ SN Ia for which they had good HST data lowered the implied Ω_m to 0.6±0.2 in the flat case [91]. Analysis of their larger dataset of 42 high-redshift SNe Ia gives for the flat case $\Omega_m = 0.28^{+0.09+0.05}_{-0.08-0.04}$ where the first errors are statistical and the second are identified systematics [92]. The High-Z Supernova team has also searched successfully for high-redshift supernovae to measure Ω_m [48, 104], and their 1998 dataset of 14 + 2 high-redshift SNe Ia including three for which they had HST data (two at $z \approx 0.5$ and one at 0.97) imply $\Omega_m = 0.32\pm0.1$ in the flat case with their MLCS fitting method.

The main concerns about the interpretation of this data are evolution of the SNe Ia [34, 106] and dimming by dust. A recent specific supernova evolution concern is that the rest frame rise-times of distant supernovae may be longer than nearby ones [105]. But a direct comparison between nearby supernova and the SCP distant sample shows that they are rather consistent with each other [2]. Ordinary dust causes reddening, but hypothetical ''grey'' dust would cause much less reddening and could in principle provide an alternative explanation for the fact that high-redshift supernovae are observed to be dimmer than expected in a critical-density cosmology. Grey interstellar dust would induce more dispersion than is observed, so the hypothetical grey dust would have to be intergalactic. It is hard to see why the largest dust grains, which would be greyer, should preferentially be ejected by galaxies [118]. Such dust, if it exists, would also absorb starlight and reradiate it at long wavelengths, where there are other constraints that could, with additional observations, rule out this scenario [1]. Such grey dust would also produce some reddening which could be detectable via comparison of infrared vs. optical colors of supernovae; such a measurement for one high-redshift SN Ia disfavors significant grey dust extinction [107], and more observations could strengthen this conclusion. Yet another way of addressing this question is to collect data on supernovae with redshift $z > 1$, where the dust scenario predicts considerably more dimming than the Λ cosmology. The one $z > 1$ supernova currently available, SCP's ''Albinoni'' (SN1998eq) at $z = 1.2$, favors the Λ cosmology. More such data are needed for a statistically significant result, and both the SCP and the High-Z group are attempting to get a few more very high redshift supernovae.

CMB anisotropies. The location of the first acoustic (or Doppler, or Sakharov) peak at angular wavenumber $l \approx 200$ indicated by the data available at the time of this meeting was evidence in favor of a flat universe $\Omega_{tot} \equiv \Omega_m + \Omega\Lambda \approx 1$ (e.g. [33]). New data from the BOOMERANG long-duration balloon flight around Antarctica [30] and the MAXIMA-1 balloon flight [54] confirm this, with $\Omega_{tot} = 1.11^{+0.13}_{-0.12}$ at 95% C.L. [59]. The preliminary BOOMERANG results [30] are lower around $l \approx 500$ than the predictions in this second peak region in ΛCDM-type models (e.g., [57]), and this could [75] indicate higher Ω_b than expected from Big Bang Nucleosynthesis together with the recent deuterium measurements (discussed below). However, the MAXIMA-1 data for $l \approx 500$ are more consistent with expectations of standard models and the standard BBN Ω_b [5] (but cf. [59]). The BOOMERANG and MAXIMA-2 data are still being analyzed, and other experiments will have relevant data as well. Further data should be available in 2001 from the NASA Microwave Anisotropy Probe satellite.

Large-scale Measurements. The comparison of the IRAS redshift surveys with POTENT and related analyses typically give values for the parameter $\beta_I \equiv \Omega_m^{0.6} / b_I$ (where b_I is the biasing parameter for IRAS galaxies), corresponding to $0.3 \lesssim \Omega_m \lesssim 3$ (for an assumed $b_I = 1.15$). It is not clear whether it will be possible to reduce the spread in these values significantly in the near future -- probably both additional data and a better understanding of systematic and statistical effects will be required. A particularly simple way to deduce a lower limit on Ω_m from the POTENT peculiar velocity data was proposed by [31], based on the fact that high-velocity outflows from voids are not expected in low-Ω models. Data on just one nearby void indicates that $\Omega_m \geq 0.3$ at the 97% C.L. Stronger constraints are available if we assume that the probability distribution function (PDF) of the primordial fluctuations was Gaussian. Evolution from a Gaussian initial PDF to the non-Gaussian mass distribution observed today requires considerable gravitational nonlinearity, i.e. large Ω_m. The PDF deduced by POTENT from observed velocities (i.e., the PDF of the mass, if the POTENT reconstruction is reliable) is far from Gaussian today, with a long positive-fluctuation tail. It agrees with a Gaussian initial PDF if and only if $\Omega_m \sim 1$; $\Omega_m < 1$ is rejected at the 2σ level, and $\Omega_m \leq 0.3$ is ruled out at $\geq 4\sigma$ [87, 10]. It would be interesting to repeat this analysis with newer data. Analyzing peculiar velocity data without POTENT again leads to a strong lower limit $\Omega_m > 0.3$ (99% C.L.), and together with the SN Ia constraints leads to the conclusion that $\Omega_m \gtrsim 0.5$ [136].

Measurements on Scales of a Few Mpc. A study by the Canadian Network for Observational Cosmology (CNOC) of 16 clusters at $z \sim 0.3$, mostly chosen from the Einstein Medium Sensitivity Survey [55], was designed to allow a self-contained measurement of Ω_m from a field M/L which in turn was deduced from their measured cluster M/L. The result was $\Omega_m = 0.19 \pm 0.06$ [18]. These data were mainly compared to standard CDM models, and they appear to exclude $\Omega_m = 1$ in such models.

Estimates on Galaxy Halo Scales. Work by Zaritsky et al. [133] has confirmed that spiral galaxies have massive halos. They collected data on satellites of isolated spiral galaxies, and concluded that the fact that the relative velocities do not fall off out to a separation of at least 200 kpc shows that massive halos are the norm. The typical rotation velocity of ~ 200 - 250 km s^{-1} implies a mass within 200 kpc of $\sim 2 \times 10^{12}\ M\odot$. A careful analysis taking into account selection effects and satellite orbit uncertainties concluded that the indicated value of Ω_m exceeds 0.13 at 90% confidence [135], with preferred values exceeding 0.3. Newer data suggesting that relative velocities do not fall off out to a separation of ~ 400 kpc [134] presumably would raise these Ω_m estimates. Weak lensing data confirms the existence of massive galactic halos [116, 125, 4, 131].

Cluster Baryons vs. Big Bang Nucleosynthesis. White et al. [128] emphasized that X-ray observations of the abundance of baryons in clusters can be used to determine Ω_m if clusters are a fair sample of both baryons and dark matter, as they are expected to be based on simulations [38]. The fair sample hypothesis implies that

$$\Omega_m = \frac{\Omega_b}{f_b} = 0.3 \left(\frac{\Omega_b}{0.04}\right) \left(\frac{0.13}{f_b}\right) . \qquad (1)$$

We can use this to determine Ω_m using the baryon abundance $\Omega_b h^2 = 0.019 \pm 0.0024$ (95% C.L.) from the measurement of the deuterium abundance in high-redshift Lyman limit systems, of which a third has recently been analyzed [66, 122] and more are in the pipeline D. Tytler, these proceedings. Using X-ray data from an X-ray flux limited sample of clusters to estimate the baryon fraction $f_b = 0.075 h^{-3/2}$ [84] gives $\Omega_m = 0.25 h^{-1/2} = 0.3 \pm 0.1$ using $h = 0.65 \pm 0.08$. Estimating the baryon fraction

using Sunyaev-Zel'dovich measurements of a sample of 18 clusters gives $f_b = 0.077h^{-1}$ [19], and implies $\Omega_m = 0.25h^{-1} = 0.38\pm0.1$.

Cluster Evolution. The dependence of the number of clusters on redshift can be a useful constraint on theories [36]. But the cluster data at various redshifts are difficult to compare properly since they are rather inhomogeneous. Using just X-ray temperature data, [37] concludes that $\Omega_m \approx 0.45\pm0.2$, with $\Omega_m = 1$ strongly disfavored.

Power Spectrum. In the context of the Λ CDM class of models, two additional constraints are available. The spectrum shape parameter $\Gamma \approx \Omega_m h \approx 0.25\pm0.05$, implying $\Omega_m \approx 0.4\pm0.1$. A new measurement $\Omega_m = 0.34\pm0.1$ comes from the amplitude of the power spectrum of fluctuations at redshift $z \sim 3$, measured from the Lyman α forest [127]. This result is strongly inconsistent with high-Ω_m models because they would predict that the fluctuations grow much more to $z = 0$, and thus would be lower at $z = 3$ than they are observed to be.

7. CONCLUSION

We thus end up with a picture of the distribution of the density of energy density in a flat universe represented by Figure 1 [99]. One of the most striking things about the present era in cosmology is the remarkable agreement between the values of the cosmological densities and the other cosmological parameters obtained by different methods - except possibly for the quasar lensing data which favors a higher Ω_m and lower $\Omega\Lambda$, and the arc lensing data which favors lower values of both parameters. If the results from the new CMB measurements end up agreeing with those from the other methods discussed above, the cosmological parameters will have been determined to perhaps 10%, and cosmologists can focus their attention on the other subjects that I mentioned at the beginning: origin of the initial fluctuations, the nature of the dark matter and dark energy, and the formation of galaxies and large-scale structure. Cosmologists can also speculate on the reasons why the cosmological parameters have the values that they do, but this appears to be the sort of question whose answer may require a deeper understanding of fundamental physics - perhaps from a superstring theory of everything.

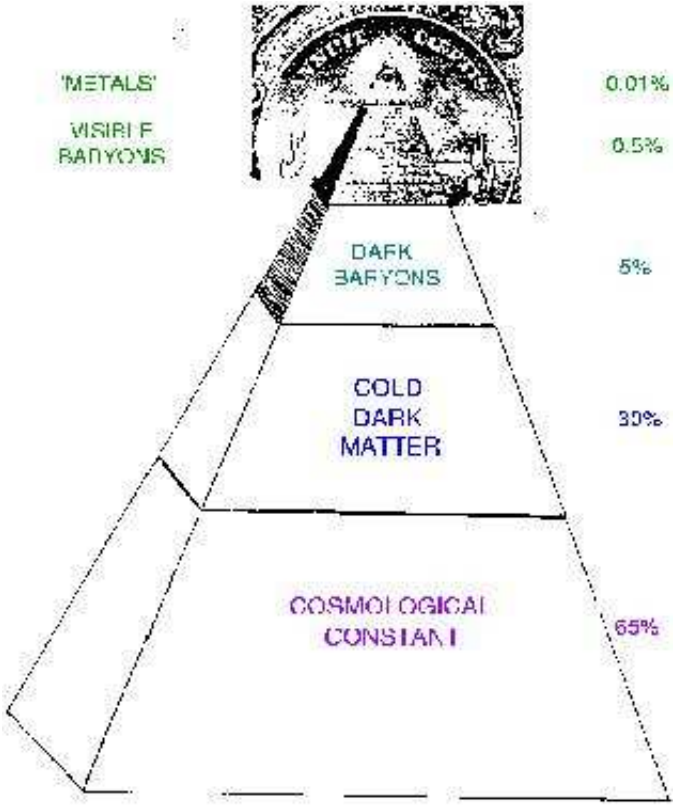

Figure 1. The Great Seal of the United States, found on the back of the American dollar bill, includes a pyramid representing strength and duration, capped by the eye of Providence. Here we use this to represent the visible matter in the universe ($\Omega_{vis} \approx 0.005$), with the upper triangle containing the eye representing the metals (elements heavier than hydrogen and helium, with $\Omega_{metals} \approx 10^{-4}$) since most of the mass of our bodies is made up of these elements. The three-dimensional nature of the pyramid, which here continues below the part shown on the Great Seal, makes it useful for showing graphically the relative proportions of the dark baryons, cold dark matter, and cosmological constant (or dark energy).

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