The cosmological constant problem and quintessence.

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Abstract. I briefly review the cosmological constant problem and the issue of dark energy (or quintessence). Within the framework of quantum field theory, the vacuum expectation value of the energy momentum tensor formally diverges as k^4. A cutoff at the Planck or electroweak scale leads to a cosmological constant which is, respectively, 10^{123} or 10^{55} times larger than the observed value, $\Lambda/8\pi G \simeq 10^{-47}$ GeV^4. The absence of a fundamental symmetry which could set the value of Λ to either zero or a very small value leads to *the cosmological constant problem*. Most cosmological scenario's favour a large time-dependent Λ-term in the past (in order to generate inflation at $z \gg 10^{10}$), and a small Λ-term today, to account for the current acceleration of the universe at $z \lesssim 1$. Constraints arising from cosmological nucleosynthesis, CMB and structure formation constrain Λ to be sub-dominant during most of the intermediate epoch $10^{10} < z < 1$. This leads to the *cosmic coincidence* conundrum which suggests that the acceleration of the universe is a recent phenomenon and that we live during a special epoch when the density in Λ and in matter are almost equal. Time varying models of dark energy can, to a certain extent, ameliorate the fine tuning problem (faced by Λ), but do not resolve the puzzle of cosmic coincidence. I briefly review tracker models of dark energy, as well as more recent brane inspired ideas and the issue of horizons in an accelerating universe. Model independent methods which reconstruct the cosmic equation of state from supernova observations are also assessed. Finally, a new diagnostic of dark energy – 'Statefinder', is discussed.

1. Introduction

Einstein (1917) introduced the cosmological constant Λ because he believed that a closed static universe which emerged in the presence of both Λ and matter agreed with Ernst Mach's concepts of inertia [1, 2] which forbade the notion of 'empty space'. However, the discovery by Friedmann (1922) of expanding solutions to the Einstein field equations in the absence of Λ, together with the discovery by Hubble (1929) that the universe was expanding, gave a blow to the static model[3, 4]. Soon after, Einstein discarded the cosmological constant. Although abandoned by Einstein, the cosmological constant staged several come-backs. It was soon realised that, since the static Einstein universe is unstable to small perturbations, one could construct expanding universe models which had a quasi-static origin in the past, thus ameliorating the initial singularity which plagues expanding FRW models. One could also construct models which approached the static Einstein universe during an intermediate epoch when the universe 'loitered' with $a \simeq$ constant. Such a model was proposed by Lemaitre (1927) and was to prove influential later, in 1968, when it was invoked to explain an alleged excess of quasars at a redshift $z \sim 2$. It is also interesting that during the very same year that Einstein proposed the cosmological constant, de Sitter discovered a matter-free solution to the Einstein equations in the presence of Λ, which had both static and dynamic representations. The de Sitter metric was to play an important role both in connection with steady state cosmology as well as in the construction of inflationary models of the very early universe.

A physical basis for the cosmological constant had to wait until 1968, when Ya. B. Zel'dovich puzzling over cosmological observations which appeared to require Λ (the quasar excess at $z \sim 2$ alluded to earlier) realised that one loop quantum vacuum fluctuations‡ gave rise to an energy momentum tensor which, after being suitably regularised for infinities, had exactly the same form as a cosmological constant: $\langle T_{ik} \rangle_{\rm vac} = \Lambda g_{ik}/8\pi G$.

Theoretical interest in Λ remained on the increase during the 1970's and early 1980's with the construction of inflationary models, in which matter (in the form of a false vacuum, as vacuum polarization or as a minimally coupled scalar-field) behaved precisely like a weakly time-dependent Λ-term. The current interest in Λ stems mainly from observations of Type Ia high redshift supernovae which indicate that the universe is accelerating fueled perhaps by a small cosmological Λ-term [10, 11]. §

2. The cosmological constant and vacuum energy

Vacuum fluctuations contributing to Λ generate a very large (formally infinite) value of the cosmological constant $\langle T_{00} \rangle_{\rm vac} \propto \int_0^\infty \sqrt{k^2+m^2}k^2 dk$. The integral diverges as k^4 resulting in an infinite value for $\langle T_{00} \rangle_{\rm vac}$ and hence also for the cosmological constant $\Lambda = 8\pi G \langle T_{00} \rangle_{\rm vac}$. Since each form of energy gravitates and therefore reacts back on the space-time geometry, an infinite value of Λ is expected to generate an infinitely large space-time curvature through the semi-classical Einstein equations $G_{00} = \frac{8\pi G}{c^4}\langle T_{00} \rangle_{\rm vac}$. One way to avoid this is to assume that the Planck scale provides a natural ultra-violet cutoff to all field theoretic processes, this results in $\langle T_{00} \rangle_{\rm vac} \simeq c^5/G^2\hbar \sim\sim 10^{76}{\rm GeV}^4$ which is 123 orders of magnitude larger than the currently observed value $\rho_\Lambda \simeq 10^{-47}{\rm GeV}^4$. A cutoff at the much lower QCD scale doesn't fare much better since it generates a cosmological constant $\Lambda_{QCD}^4 \sim 10^{-3}{\rm GeV}^4$ – forty orders of magnitude larger than observed. Clearly the answer to the cosmological constant issue must lie elsewhere.

The discovery of supersymmetry in the 1970's led to the hope that the cosmological constant problem may be resolved by a judicious balance between bosons and fermions in nature, since bosons and fermions (of identical mass) contribute equally but with opposite sign to the vacuum expectation value of physical quantities,

‡ The presence of zero-point vacuum fluctuations was predicted by Casimir [8] and has been verified by several experiments, see [9] and references therein.

§ The chronology of interest in Λ bears a curious historical parallel to the scientific fascination with the notion of extra dimensions. (I thank Nathalie Duruelle for an interesting discussion on this issue during the meeting.) A fourth spatial dimension was proposed by Nordström (1914) and independently by Kaluza (1921), but the real scientific interest in higher dimensions grew after de Witt (1962), Kerner (1968) and others [16] had convincingly demonstrated the deep relationship between higher dimensional theories on the one hand, and non-abelian gauge fields on the other. Cosmology in a space-time with extra dimensions really took off during the 1970's and early 1980's when grand unified and supergravity models frequently relied on compact extra dimensions to generate the extra gauge degrees of freedom associated with unification. Current interest in higher dimensional cosmologies is spurred by superstring theory as well as by 'brane-world' scenario's of extra dimensions.

so that

$$\langle 0|H_{\mathrm{b,f}}|0\rangle \equiv \int dV \langle T_{00}\rangle_{\mathrm{vac}} = \pm\frac{1}{2}\sum_{\mathbf{k}} \omega_{\mathbf{k}}. \quad (1)$$

However supersymmetry (if it exists) is broken at the low temperatures prevailing in the universe today and on this account one should expect the cosmological constant to vanish in the early universe, but to reappear during late times when the temperature has dropped below T_{SUSY}. This is clearly an undesirable scenario and almost the very opposite of what one is looking for, since, a large value of Λ at an early time is useful from the viewpoint of inflation, whereas a very small current value of Λ is in agreement with observations.

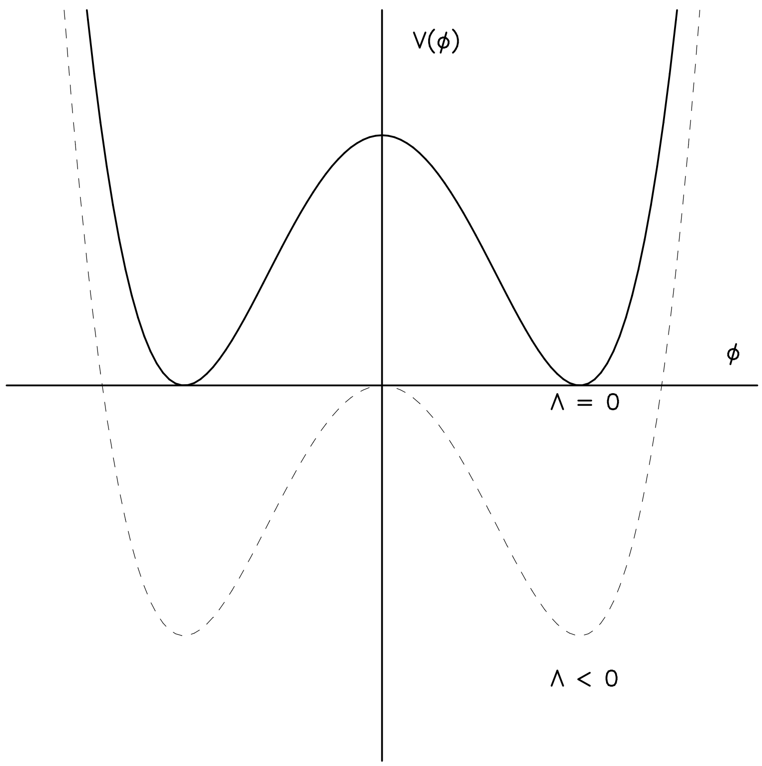

Figure 1. The potential describing spontaneous symmetry breaking has the form of a Mexican top hat. The dashed line shows the potential before the cosmological constant has been 'renormalized' and the solid line after.

The cosmological constant also makes an important appearance in models with spontaneous symmetry breaking [17]. For simplicity consider the Lagrangian

$$\begin{aligned} \mathcal{L} &= \frac{1}{2}\, g^{ij}\partial_i\phi\partial_j\phi - V(\phi), \\ V(\phi) &= V_0 - \frac{1}{2}\mu^2\phi^2 + \frac{1}{4}\lambda\phi^4. \end{aligned} \quad (2)$$

The symmetric state at $\phi = 0$ is unstable and the system settles in the ground state $\phi = +\sigma$ or $\phi = -\sigma$, where $\sigma = \sqrt{\mu^2/\lambda}$, thus breaking the reflection symmetry

$\phi \leftrightarrow -\phi$ present in the Lagrangian. If $V_0 = 0$ then this potential results in a broken symmetry state with a large negative cosmological constant $\Lambda_{eff} = V(\phi = \sigma) = -\mu^4/4\lambda$. In order to avoid this situation the value of the free parameter V_0 is chosen to counterbalance Λ_{eff}, as a result one sets $V_0 \sim \mu^4/4\lambda$ so that $\Lambda_{eff}/8\pi G = V_0 - \mu^4/4\lambda \simeq 10^{-47}\mathrm{GeV}^4$. The ensuing 'regularization' of the large negative cosmological constant must be done with considerable care, since even small 'fluctuations' in the final value of Λ can result in grave consequences for cosmology. For instance if $\Lambda_{eff}/8\pi G < -10^{-43}\mathrm{GeV}^4$ the large attractive force exerted by a negative cosmological constant will ensure that the universe re-collapses before it reaches 'maturity'. The age of the universe in this case will be < 1 billion years, much too short for galaxies to form and for life (as we know it) to emerge within the standard big bang scenario. On the other hand if $\Lambda_{eff}/8\pi G > 10^{-43}\mathrm{GeV}^4$, the large repulsive force generated by Λ will ensure that the universe begins accelerating before gravitationally bound systems have a chance to form. Such a scenario will also clearly preclude the emergence of life.

The rather small window permitted for life to emerge in the presence of Λ has led several researchers [18, 19, 20] to develop anthropic arguments for the existence of a small cosmological constant. A possibility which is summarised by the following sentence: "if our big bang is just one of many big bangs, with a wide range of vacuum energies, then it is natural that some of these big bangs should have a vacuum energy in the narrow range where galaxies can form, and of course it is just these big bangs in which there could be astronomers and physicists wondering about the vacuum energy" [20]. Anthropic arguments for Λ will not be examined further by me in this talk.

In the absence of a fundamental symmetry of nature which will set the value of Λ to zero one has to look towards physical mechanisms which might generate an acceptably small value of the Λ-term today.

Exploring the connection between quantum fluctuations and Λ Zel'dovich suggested that, after the removal of divergences, the energy density of a virtual particle-antiparticle pair interacting gravitationally would be [7]

$$\rho_\Lambda \sim \frac{Gm^2}{\hbar c} m \, (\frac{mc}{\hbar})^3. \tag{3}$$

(This result is easy to derive if one notes that the interaction energy density is typically $\epsilon_{vac} \equiv \rho_{vac} c^2 \sim \frac{Gm^2}{\lambda}/\lambda^3$ where $\lambda = \hbar/mc$ is the mean separation between particle and antiparticle.) This possibility has not been explored much, perhaps because the proton-antiproton (electron-positron) contribution gives a very large (small) value for ρ_Λ. Interestingly the pion-antipion mass gives just the right value $\rho_\Lambda = \frac{1}{(2\pi)^4} \, \rho_{\rm Pl} \, (m_\pi/M_{\rm Pl})^6 \simeq 1.3 \times 10^{-123} \rho_{\rm Pl} = 6.91 \times 10^{-30} \; g\, cm^{-3}$.

Purely numerological considerations also allow one to generate a sufficiently small value of Λ through a suitable combination of fundamental constants. For instance the fine structre constant α can be combined with the Planck density $\rho_{\rm Pl}$ to give [21] $\rho_\Lambda = \frac{\rho_{\rm Pl}}{(2\pi^2)^3} e^{-2/\alpha} \simeq 1.2 \times 10^{-123} \rho_{\rm Pl} = 6.29 \times 10^{-30} \; g\, cm^{-3}$.

A small vacuum energy may be connected to fundamental physics in other (equally speculative) ways. It is interesting that the mass scale associated with the scale of supersymmetry breaking in some models, $M_{\rm SUSY} \sim 1$ TeV, lies midway between the Planck scale and 10^{-3} eV. The small observed value of the cosmological constant $\rho_\Lambda \simeq (10^{-3} eV)^4$ might therefore be associated with the vacuum in a theory which had a fundamental mass scale $M_X \simeq M_{\rm SUSY}^2/M_{\rm Pl}$, such that $\rho_{\rm vac} \sim M_X^4 \sim (10^{-3} eV)^4$.

3. A dynamical Λ-term.

Any fundamental theory of nature which intends to succesfully generate Λ will be confronted by the 'fine tuning problem' since the currently observed value of the cosmological constant is miniscule when compared with either the Planck ($\rho_\Lambda/M_{\rm Pl}^4 \sim 10^{-123}$) or the electroweak scale ($\rho_\Lambda/M_{\rm EW}^4 \sim 10^{-55}$). During the expansion of the universe the energy density in matter (radiation) decreases as a^{-3} (a^{-4}) while the density in Λ remains constant. As a result an enormous fine tuning of initial conditions is required in order to ensure that the cosmological Λ-term comes to dominate the expansion dynamics of the universe at precisely the current epoch, no sooner and no later.

The fine-tuning problem is rendered less acute if we relax the condition ρ_Λ =constant, and (taking the cue from Inflation) try to construct dynamical models for ρ_Λ.

Phenomenological approaches to a dynamical Λ-term belong to three main categories [21]:

(1) *Kinematic models.*

ρ_Λ is simply assumed to be a function of either the cosmic time t or the scale factor $a(t)$ of the FRW cosmological model.

(2) *Hydrodynamic models.*

ρ_Λ is described by a barotropic fluid with some equation of state $p_\Lambda(\rho_\Lambda)$ (dissipative terms may also be present).

(3) *Field-theoretic models.* The Λ-term is assumed to be a new physical classical field with some phenomenological Lagrangian.

The simplest class of kinematic models

$$\Lambda \equiv 8\pi G\rho_\Lambda = f(a) \tag{4}$$

is equivalent to hydrodynamic models based on an ideal fluid with an equation of state

$$p_\Lambda(\rho_\Lambda) = -\rho_\Lambda(1 + \frac{1}{3}\frac{d\ln\rho_\Lambda}{d\ln a}). \tag{5}$$

The expansion of the universe passes through an inflection point the moment it stops decelerating and begins to accelerate. If the equation of state is held constant ($w = P/\rho < -1/3$) then the cosmological redshift when this occurs is given by

$$(1 + z_{\rm a})^{-3w} = -(1 + 3w)\frac{\Omega_X}{\Omega_m}. \tag{6}$$

We find that $z_{\rm a} \simeq 0.7$ for the cosmological constant ($w = -1$) with $\Omega_\Lambda \simeq 0.7$ and $\Omega_m \simeq 0.3$. The acceleration of the universe is therefore a *very recent* phenomena. This fact is related to the *cosmic coincidence* conundrum since it appears that we live during a special era when the density of dark matter and dark energy are comparable. The cosmic coincidence puzzle remains in place even if we relax the assumption $w = -1$ and allow dark energy to be time dependent. Indeed, it is easy to show that the equality between dark matter and dark energy takes place at $(1 + z_{\rm eq})^3 = (\Omega_\Lambda/\Omega_m)^{-1/w}$. For a cosmological constant this gives $z_{\rm eq} \simeq 0.3$ and $z_{\rm a} > z_{\rm eq}$ implying that the universe begins to accelerate even *before* it becomes Λ-dominated. For $w = -2/3$ $z_{\rm a} = z_{\rm eq} \simeq 0.5$, while for stiffer equations of state $z_{\rm a} < z_{\rm eq}$ ($w > -2/3$) further exacerbating the cosmological coincidence puzzle.

3.1. Scalar field models of dark energy

Although the *cosmic coincidence* issue remains unresolved, the fine tuning problem facing dark energy/quintessence models with a constant equation of state can be significantly alleviated if we assume that the equation of state is time dependent. An important class of models having this property are scalar fields which couple minimally to gravity and whose energy momentum tensor is

$$\rho \equiv T_0^0 = \frac{1}{2}\dot{\phi}^2 + V(\phi),\ P \equiv -T_\alpha^\alpha = \frac{1}{2}\dot{\phi}^2 - V(\phi). \tag{7}$$

A scalar field rolling down its potential *slowly* generates a time-dependent Λ-term since $P \simeq -\rho \simeq -V(\phi)$ if $\dot{\phi}^2 \ll V(\phi)$. Potentials which satisfy $\Gamma \equiv V''V/(V')^2 \geq 1$ have the interesting property that scalar fields approach a common evolutionary path from a wide range of initial conditions [22]. In these so-called 'tracker' models the scalar field density (and its equation of state) remains close to that of the dominant background matter during most of cosmological evolution. A good example is provided by the exponential potential $V(\phi) = V_0 \exp\left(-\sqrt{8\pi}\lambda\phi/M_{\rm Pl}\right)$ [23, 24] for which

$$\frac{\rho_\phi}{\rho_B + \rho_\phi} = \frac{3(1+w_B)}{\lambda^2} = \text{constant} < 0.2, \tag{8}$$

ρ_B is the background energy density while w_B is the associated equation of state. The lower limit $\rho_\phi/\rho_{\rm total} < 0.2$ arises because of nucleosynthesis constraints which prevent the energy density in quintessence from being large initially (at $t \sim few$ sec.). Since the ratio $\rho_\phi/\rho_{\rm total}$ remains fixed, exponential potentials on their own cannot supply us with a means of generating dark energy/quintessence at the present epoch. However a suitable modification of the exponential achieves this. For instance the class of potentials [26]

$$V(\phi) = V_0[\cosh\lambda\phi - 1]^p, \tag{9}$$

has the property that $w_\phi \simeq w_B$ at early times whereas $\langle w_\phi\rangle = (p-1)/(p+1)$ at late times. Consequently (9) describes *quintessence* for $p \leq 1/2$ and pressureless 'cold' dark matter (CDM) for $p = 1$.

A second example of a tracker-potential is provided by $V(\phi) = V_0/\phi^\alpha$ [23]. During tracking the ratio of the energy density of the scalar field (quintessence) to that of radiation/matter gradually increases $\rho_\phi/\rho_B \propto t^{4/(2+\alpha)}$ while its equation of state remains marginally smaller than the background value $w_\phi = (\alpha w_B - 2)/(\alpha+2)$. These properties allow the scalar field to eventually dominate the density of the universe, giving rise to a late-time epoch of accelerated expansion. (Current observations place the strong constrain $\alpha \lesssim 2$.)

Several of the quintessential potentials listed in table 1 have been inspired by field theoretic ideas including supersymmetric gauge theories and supergravity, pseudo-goldstone boson models, etc. However accelerated expansion can also arise in models with: (i) topological defects such as a frustrated network of cosmic strings ($w \simeq -1/3$) and domain walls ($w \simeq -2/3$)[31]; (ii) scalar field lagrangians with non-linear kinetic terms and no potential term (k-essence [32]); (iii) vacuum polarization associated with an ultra-light scalar field [33, 34]; (iv) non-minimally coupled scalar fields [35]; (v) fields that couple to matter [36]; (vi) scalar-tensor theories of gravity [37]; (vii) brane-world models [38, 39, 40, 41, 43, 44] etc.

Scalar field based quintessence models can be broadly divided into two classes: (i) those for which $\phi/M_{\rm Pl} \ll 1$ as $t \to t_0$, (ii) those for which $\phi/M_{\rm Pl} \geq 1$ as $t \to t_0$

Quintessence Potential	Reference
$V_0 \exp(-\lambda\phi)$	Ratra & Peebles (1988), Wetterich (1988), Ferreira & Joyce (1998)
$m^2\phi^2, \lambda\phi^4$	Frieman et al (1995)
$V_0/\phi^\alpha, \alpha > 0$	Ratra & Peebles (1988)
$V_0 \exp(\lambda\phi^2)/\phi^\alpha, \alpha > 0$	Brax & Martin (1999,2000)
$V_0(\cosh \lambda\phi - 1)^p$,	Sahni & Wang (2000)
$V_0 \sinh^{-\alpha}(\lambda\phi)$,	Sahni & Starobinsky (2000), Ureña-López & Matos (2000)
$V_0(e^{\alpha\kappa\phi} + e^{\beta\kappa\phi})$	Barreiro, Copeland & Nunes (2000)
$V_0(\exp M_p/\phi - 1)$,	Zlatev, Wang & Steinhardt (1999)
$V_0[(\phi - B)^\alpha + A]e^{-\lambda\phi}$,	Albrecht & Skordis (2000)

Table 1.

(t_0 is the present time). An important issue concerning the second class of models is whether quantum corrections become important when $\phi/M_{\rm Pl} \geq 1$ and their possible effect on the quintessence potential [45]. One can also ask whether a given choice of parameter values is 'natural'. Consider for instance the potential $V = M^{4+\alpha}/\phi^\alpha$, current observations indicate $V_0 \simeq 10^{-47}{\rm GeV}^4$ and $\alpha \lesssim 2$, which together suggest $M \lesssim 0.1$ GeV (smaller values of M arise for smaller α) it is not clear whether such small parameter values can be motivated by current models of high energy physics.

Finally, it would be enormously interesting if one and the same field could give rise to both Inflation and dark energy. Such models have been discussed both in the context of standard inflation [46] and brane-world inflation [39, 40, 41], we briefly discuss the second possibility below.

3.2. Quintessential Inflation in Braneworld scenario's

In the 4+1 dimensional brane scenario inspired by the Randall-Sundrum [47] model, matter fields are confined to a three dimensional 'brane' which is embedded in a four dimensional 'bulk' geometry. The equation of motion of a scalar field propogating on the brane is

$$\ddot{\phi} + 3H\dot{\phi} + V'(\phi) = 0. \tag{10}$$

where [48]

$$H^2 = \frac{8\pi}{3M_4^2}\rho(1 + \frac{\rho}{2\lambda_b}),\ \rho = \frac{1}{2}\dot{\phi}^2 + V(\phi), \tag{11}$$

and λ_b is the brane tension. The additional term ρ^2/λ_b in (11) arises due to junction conditions imposed at the bulk-brane boundary. The presence of this term *increases the damping* experienced by the scalar field as it rolls down its potential. This effect is reflected in the slow-roll parameters, which in braneworld models (for $V/\lambda_b \gg 1$) have the form [38]

$$\epsilon \simeq 4\epsilon_{\rm FRW}(V/\lambda_b)^{-1},\ \eta \simeq 2\eta_{\rm FRW}(V/\lambda_b)^{-1}. \tag{12}$$

Clearly slow-roll ($\epsilon, \eta \ll 1$) is easier to achieve when $V/\lambda_b \gg 1$ and on this basis one can expect inflation to occur even for the very steep potentials associated with quintessence models including $V \propto e^{\lambda\phi}$, $V \propto \phi^{-\alpha}$ etc. Inflation in these models has been extensively discussed in [39, 40, 41, 42] within the framework of a scenario in which reheating takes place unconventionally, through inflationary particle production. This leads to an enormous difference between the energy in the inflaton and in radiation at the end of inflation: $\rho_\phi/\rho_{rad}|_{\rm end} \sim 10^{16}$. Since the potential driving inflation is steep, the post-inflationary expansion in these models is driven by the kinetic energy of the scalar field, so that $w_\phi \simeq 1$, $\rho_\phi \propto a^{-6}$ and $a \propto t^{1/3}$. (Because radiation decreases at the slower rate $\rho_{\rm rad} \propto a^{-4}$ the scale factor changes to $a \propto t^{1/2}$ after the density in the inflaton and in radiation equalize. This usually takes place at a low temperature $T_{\rm eq} \sim$ few GeV.)

As demonstrated in [39, 40, 41, 42] inflation can occur for several of the quintessence potentials discussed in the previous section but for a rather narrow region of parameter space (see figure 2). It also appears that quintessential inflation generates a large gravity wave background which could be in conflict with big bang nucleosynthesis considerations [41].

3.3. Reconstructing the cosmic equation of state

Although fundamental theories such as Supergravity or M-theory do provide a number of possible candidates for quintessence they do not uniquely predict its potential $V(\phi)$. Therefore it becomes meaningful to reconstruct $V(\phi)$ and the cosmic equation of state $w = P/\rho$ directly from observations in a *model independent* manner [49, 50, 51, 52]. This is possible to do if one notices that the scalar field potential as well as its equation of state can be directly expressed in terms of the Hubble parameter and its derivative

$$\frac{8\pi G}{3H_0^2}V(x) = \frac{H^2}{H_0^2} - \frac{x}{6H_0^2}\frac{dH^2}{dx} - \frac{1}{2}\Omega_m\, x^3, \tag{13}$$

$$\frac{8\pi G}{3H_0^2}\left(\frac{d\phi}{dx}\right)^2 = \frac{2}{3H_0^2 x}\frac{d\ln H}{dx} - \frac{\Omega_m x}{H^2},\ x = 1+z, \tag{14}$$

$$\tag{15}$$

$$w_\phi(x) \equiv \frac{p}{\varepsilon} = \frac{(2x/3)d\ln H/dx - 1}{1 - (H_0^2/H^2)\,\Omega_m x^3}. \tag{16}$$

Since the Hubble parameter is related to the luminosity distance

$$H(z) = \left[\frac{d}{dz}\left(\frac{d_L(z)}{1+z}\right)\right]^{-1}, \tag{17}$$

one can determine both the quintessence potential $V(\phi)$ as well as reconstruct its equation of state $w_\phi(z)$ provided the luminosity distance $d_L(z)$ is known from observations. A three parameter ansatz for estimating the luminosity distance was proposed in [49]. Results from that paper reproduced in figure 3 indicate that only a

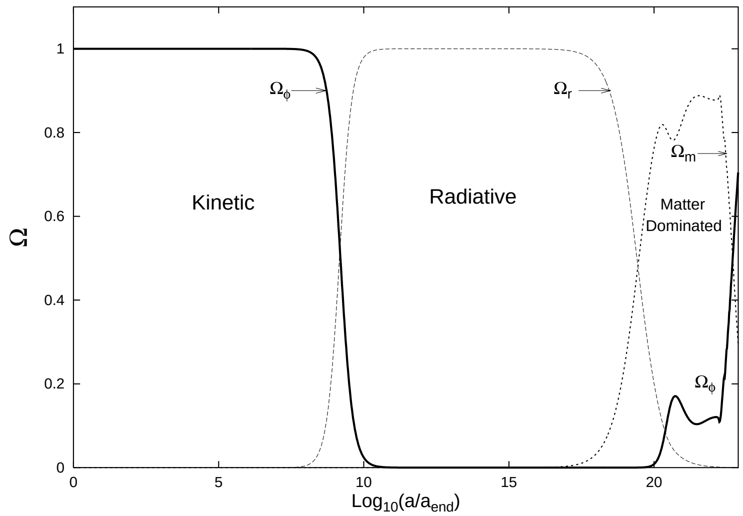

Figure 2. The post-inflationary density parameter Ω is plotted for the scalar field (solid line) radiation (dashed line) and cold dark matter (dotted line) in the quintessential-inflationary model decribed by (9) with $p = 0.2$. Late time oscillations of the scalar field ensure that the mean equation of state turns negative $\langle w_\phi \rangle \simeq -2/3$, giving rise to the current epoch of cosmic acceleration with $a(t) \propto t^2$ and present day values $\Omega_{0\phi} \simeq 0.7, \Omega_{0m} \simeq 0.3$. From Sahni, Sami and Souradeep [41].

small amount of evolution in $w_\phi(z)$ is permitted by current SnIa observations. The presence of a cosmological constant is therefore in good agreement with these results.

A word of caution should be added: as shown in figure 4 a near degeneracy exists between the equation of state of dark energy and the value of Ω_m. The latter should therefore be known to better than 5% accuracy for the reconstruction program to yield very accurate results (see also [51]).

3.4. Probing dark energy using the Statefinder statistic

An issue of the utmost importance is whether dark energy (equivalently quintessence) is a cosmological constant or whether it has a fundamentally different origin. A new dimensionless statistic 'Statefinder', recently introduced by Sahni, Saini, Starobinsky and Alam [53] has the power to discriminate between different forms of dark energy and may therefore be a good diagnostic of cosmological models.

The Statefinder pair $\{r, s\}$ is constructed from the scale factor of the universe and its derivatives and probes the cosmic equation of state and its rate of change. It extends the hierarchy of geometrical cosmological parameters to four: $\{H, q, r, s\}$, where $H = (\dot{a}/a)$, $q = -H^{-2}(\ddot{a}/a)$,

$$\begin{aligned} r &= \frac{\dddot{a}}{aH^3} = 1 + \frac{9w}{2}\Omega_\phi(1+w) - \frac{3}{2}\Omega_\phi \frac{\dot{w}}{H}, \\ s &= \frac{r-1}{3(q-1/2)} = 1 + w - \frac{1}{3}\frac{\dot{w}}{wH}. \end{aligned} \tag{18}$$

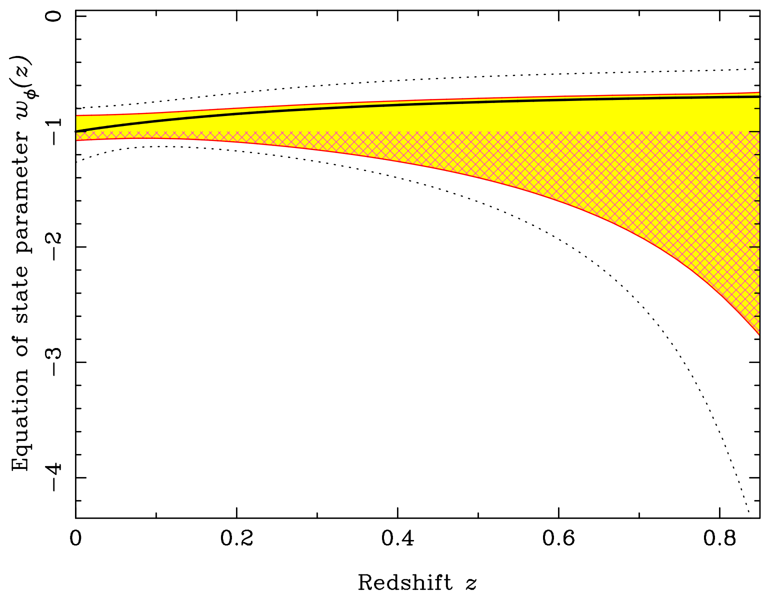

Figure 3. The equation of state of dark energy/quintessence is reconstructed from observations of Type Ia high redshift supernovae in a model independent manner. The equation of state satisfies $-1 \leq w_\phi \leq -0.8$ at $z = 0$; and $-1 \leq w_\phi \leq -0.46$ at $z = 0.83$ (90% CL), $\Omega_m = 0.3$ is assumed. From Saini, Raychaudhury, Sahni and Starobinsky [49].

From figure 5 we see that while r *remains fixed* at $r = 1$ in a universe containing matter and a cosmological constant, the value of r decreases steadily for time varying forms of dark energy. The Statefinder statistic can therefore help differentiate a cosmological constant from: (i) dark energy with a *time-independent* equation of state (referred to in [53] as Quiessence) and (ii) dark energy with a time-dependent equation of state (referred to in [53] as Kinessence).

4. Horizons in a Λ-dominated universe

The conventional viewpoint regarding the future evolution of a matter-dominated universe can be summarised by the following pair of statements:

[A] If the universe is spatially open or flat then it will expand for ever.

[B] Alternatively, if the universe is spatially closed then its expansion will be followed by recollapse.

In a Λ-dominated universe [A] and [B] are replaced by

[A'] A spatially open or flat universe ($\kappa = 0, -1$) will recollapse if the Λ-term is constant and negative.

[B'] A closed universe ($\kappa = +1$) with a constant positive Λ-term can, under certain circumstances, expand forever.

Recent CMB observations indicate that our universe is close to being spatially flat, therefore, if we wait long enough we will find that the expansion of our universe rapidly approaches the de Sitter value $H = H_\infty = \sqrt{\Lambda/3} = H_0\sqrt{1-\Omega_m}$, while the

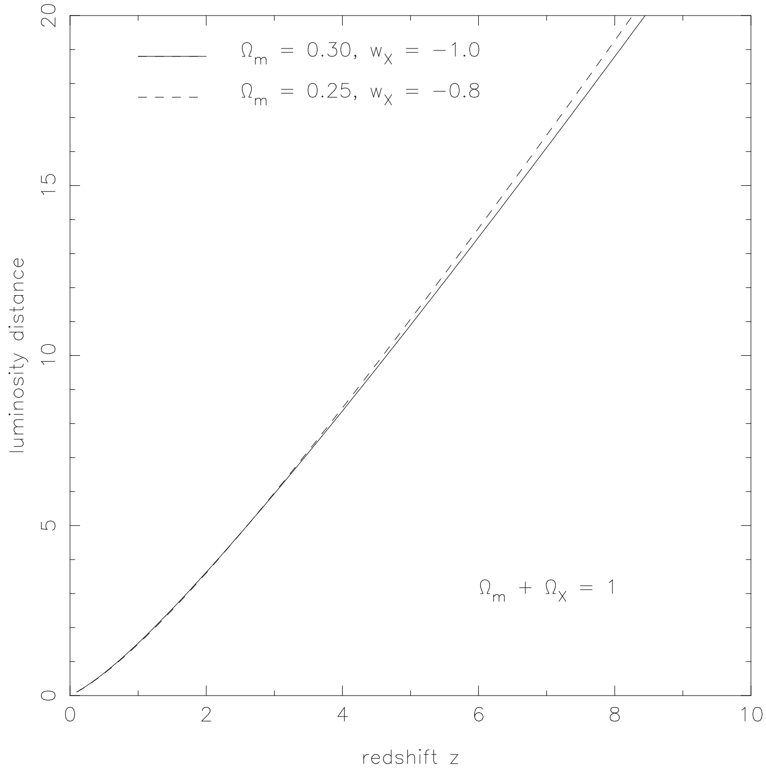

Figure 4. The near degeneracy in the luminosity distance is shown for the pair of cosmological models with $\{\Omega_m = 0.3, w_X = -1.0\}$ and $\{\Omega_m = 0.25, w_X = -0.8\}$.

density of matter asymptotically declines to zero $\rho_m \propto a^{-3} \to 0$.

Density perturbations in such a universe will freeze to a constant value if they are still in the linear regime, but the acceleration of the universe will not affect gravitationally bound systems on present scales of $R < 10\mathrm{h}^{-1}$ Mpc (which includes our own galaxy as well as galaxy clusters). The universe at late times will therefore consist of islands of matter immersed in an accelerating sea of dark energy: 'Λ'.

In such a universe the local neighborhood of an observer from which he/she is able to receive signals will eventually contract and shrink so that even those regions of the universe which are currently observable to us will eventually be hidden from view. As an illustration consider an event at (r_1, t_1) which we wish to observe at our location at $r = 0$, then

$$\int_0^{r_1} \frac{dr}{\sqrt{1 - \kappa r^2}} = \int_{t_1}^{t} \frac{dt'}{a(t')}. \tag{19}$$

An observer at $r = 0$ will be able to receive signals from any event (after a suitably long wait) provided the integral in the RHS of (19) diverges (as $t \to \infty$). For $a \propto t^p$, this implies $p < 0$, or a decelerating universe. In an accelerating universe the integral converges, signaling the presence of an event horizon. As a result one can only receive

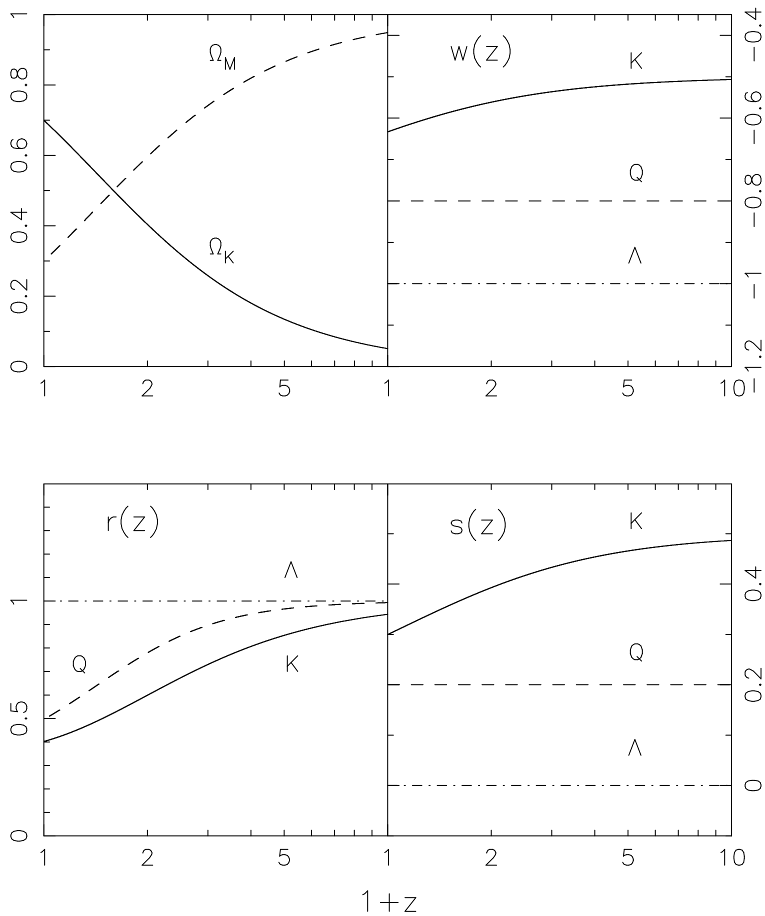

Figure 5. The Statefinder pair $\{r, s\}$ is shown for dark energy consisting of a cosmological constant Λ, Quiessence 'Q' with an unevolving equation of state $w = -0.8$ and the inverse power law tracker model $V = V_0/\phi^2$, referred to as Kinessence 'K'. The lower left panel shows $r(z)$ while the lower right panel shows $s(z)$. Kinessence has a time-dependent equation of state which is shown in the top right panel. The fractional density in matter and Kinessence is shown in the top left panel. The ability of the Statefinder pair $\{r, s\}$ to differentiate between the different forms of dark energy is amply demonstrated by this figure which is reproduced from Sahni, Saini, Starobinsky and Alam [53].

signals from those events which satisfy

$$\int_0^{r_1} \frac{dr}{\sqrt{1 - \kappa r^2}} \leq \int_{t_1}^{\infty} \frac{dt'}{a(t')}. \tag{20}$$

For de Sitter-like expansion $a = a_1 \exp H(t - t_1), H = \sqrt{\Lambda/3}$, we get $r_1 \equiv r_H$ and $R = a_1 r_H = H^{-1}$ where R is the proper distance to the event horizon. In such a universe light emitted by distant objects gets redshifted and declines in intensity (an analogous situation occurs for an object falling through the horizon of a black hole.) As a result comoving observers once visible, gradually disappear from view as the universe accelerates under the influence of Λ. One consequence of this interesting phenomenon is that at any given instant of time, t_0, one can determine a redshift z_H, which will define for us the 'sphere of influence' of our civilization. Celestial objects with $z > z_H$ will always remain inaccessible to signals emitted by our civilization at $t \geq t_0$. For a Λ-dominated universe with $\Omega_m = 0.3$, $\Omega_\Lambda = 0.7$ one finds $z_H = 1.80$ [21]. More generally, horizon's exist in a universe which begins to perpetually accelerate after a given point of time [54, 55]. (To this general category belong models of dark energy with equation of state $-1 < w < -1/3$, as well as 'runaway scalar fields' [56] which satisfy $V, V', V'' \to 0$ and $V'/V, V''/V \to 0$ as $\phi \to \infty$.) Since the conventional S-matrix approach may not work in a universe with an event horizon, such a cosmological model may pose a serious challenge to a fundamental theory of interactions such as string theory. Possible ways of cutting short 'eternal acceleration' (thereby avoiding horizons) involve scalar fields with non-monotonic potentials. For instance a flat potential with a local minimum will have a negative equation of state during slow roll, which will increase to non-negative values after the cessation of slow roll and the commencement of oscillations (provided the potential is sufficiently steep in the neighborhood of the minimum). An example is provided by a massive scalar field $V(\phi) = m^2\phi^2/2$ for which the epoch of accelerated expansion is a transient. Other possibilities are discussed in [55, 57, 43, 44, 58, 59].

5. Conclusions

One of the major concerns of cosmology today is the nature of dark energy (quintessence). While a cosmological constant appears to be the simplest option, formidable fine-tuning problems which confront the latter have led to theoretical models being developed in which both the dark energy as well as its equation of state are functions of time. Type Ia supernovae currently provide the strongest evidence for dark energy. The very recent observations of a supernova at $z \simeq 1.7$ appear to rule out simple extinction and evolution effects as major causes for the diminishing light flux from these high-z objects. It therefore appears that the dark energy is 'real' and that the universe was decelerating at $z > 0.5$ [60].

The observational situation is likely to improve as results from both deep and extensive galaxy and galaxy cluster survey's come in. It is well known that the presence of a cosmological constant changes the shape of the two point galaxy-galaxy correlation function by increasing the strength of clustering on large scales [61]. Additionally, since Λ slows down the growth of gravitational clustering, galaxy clusters are expected to be more abundant at high redshift in ΛCDM than in the standard cold dark matter scenario (SCDM). (Tracker fields can give rise to a large smooth component of matter at high z, as a result gravitational clustering takes place at a slower rate in tracker-quintessence models than in ΛCDM [62].) Both these effects are likely to be tested in the near future [63]. Indeed, recent results which measure the gravitational clustering of over 100,000 galaxies in the 2dF survey, determine a matter power spectrum which is consistent with ΛCDM and inconsistent with SCDM [64].

Combined results from CMB probes (MAP, PLANCK), galaxy surveys (2dF, SDSS, DEEP), weak lensing statistics and the possibility of a dedicated supernova telescope (SNAP) give rise to expectations that the nature of dark energy will be understood (at least at the phenomenological level) in the not too distant future.

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