The Large-Scale Velocity Field

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1 Bulk Flows as a Cosmological Probe

Hubble's Law, now spectacularly confirmed by the work of [27], [35], and [39], tells us that the distances of galaxies are proportional to their observed recession velocities, at least at low redshifts:

$$cz = H_0 r \quad . \tag{1}$$

However, this is not exactly correct. Galaxies have *peculiar velocities* above and beyond the Hubble flow indicated by Eq. (1). We denote the peculiar velocity $\mathbf{v}(\mathbf{r})$ at every point in space; the observed redshift in the rest frame of the Local Group is then:

$$cz = H_0 r + \hat{\mathbf{r}} \cdot (\mathbf{v}(\mathbf{r}) - \mathbf{v}(\mathbf{0})) \quad , \tag{2}$$

where the peculiar velocity of the Local Group itself is $\mathbf{v}(\mathbf{0})$, and $\hat{\mathbf{r}}$ is the unit vector to the galaxy in question. In practice, we will measure distances in units of km s^{-1}, which means that $H_0 \equiv 1$, and the uncertainties in the value of H_0 discussed by Freedman and Tammann in this volume are not an issue. Thus measurements of redshifts cz, and of redshift-independent distances via standard candles, yield estimates of the radial component of the velocity field.

What does the resulting velocity field tell us? On scales large enough that the rms density fluctuations are small, the equations of gravitational instability can be linearized, yielding a direct proportionality between the divergence of the velocity field and the density field at late times [33], [34]:

$$\nabla \cdot \mathbf{v}(\mathbf{r}) = -\Omega^{0.6} \delta(\mathbf{r}) \quad . \tag{3}$$

This equation is easily translated to Fourier space:

$$i\mathbf{k} \cdot \tilde{\mathbf{v}}(\mathbf{k}) = -\Omega^{0.6} \tilde{\delta}(\mathbf{k}) \quad , \tag{4}$$

which means that if we define a *velocity* power spectrum $P_v(k) \sim \left\langle \tilde{\mathbf{v}}^2(\mathbf{k}) \right\rangle$ in analogy with the usual density power spectrum $P(k)$, we find that

$$P_v(k) = \Omega^{1.2} k^{-2} P(k) \quad . \tag{5}$$

There are several immediate conclusions that we can draw from this. Peculiar velocities are tightly coupled to the *matter* density field $\delta(\mathbf{r})$. Therefore, peculiar velocities are a probe of the *matter* power spectrum; any bias of the distribution of galaxies relative to that of matter is not an issue. Moreover, Eq. (5) shows that it is in principle easier to probe large spatial scales with peculiar velocities than with the density field, because of the two extra powers of k weighting for the velocity power spectrum.

Eq. (3) shows that a comparison of the velocity field with the *galaxy* density field $\delta_{\rm gal}$ allows a test of gravitational instability theory. However, in order to do this, one must assume a relation between the galaxy density field (which is observed via redshift surveys) and the mass density field (which does the gravitating). The simplest and most common assumption (other than simply assuming the two are identical) is that they are proportional (*linear biasing*), i.e., $\delta_{\rm gal} = b\,\delta$. If this is the case, then we can rewrite Eq. (3) to give:

$$\nabla \cdot \mathbf{v}(\mathbf{r}) = -\frac{\Omega^{0.6}}{b} \delta_{\rm gal}(\mathbf{r}) \equiv -\beta\, \delta_{\rm gal}(\mathbf{r}) \quad . \tag{6}$$

Thus if the observed density and velocity field are consistent with one another and Eq. (6), we can hope to measure β. This, and other approaches to Ω via peculiar velocities are reviewed in Dekel's contribution to this volume; cf., the reviews by [11] and [45].

2 The Predicted Large-Scale Velocity Field: The Theorist's View

If a theorist is asked what the large-scale velocity field should look like, she will use the results derived above to calculate the expected amplitude of the bulk flow $v(R)$ averaged over a scale R:

$$\left\langle v(R)^2 \right\rangle = \frac{\Omega^{1.2}}{2\,\pi^2} \int \mathrm{d}k\, P(k) \widetilde{W}^2(kR) \quad , \tag{7}$$

where $\widetilde{W}$ is the Fourier Transform of the smoothing window. It is straightforward to calculate this quantity as a function of scale for any given power spectrum (cf., Fig. 9 of [44]), but going the other way is more difficult. If the phases of the Fourier modes of the density field are random, then each component of the velocity field has a Gaussian distribution, which means that $v(R)$ has a *Maxwellian* distribution; Fig. 1 reminds us just how broad such a distribution is. Therefore, a single measurement of the bulk flow on large scales gives us a relatively weak handle on the power spectrum.

How then can we constrain the observed power spectrum with observations of the velocity field? Under the random phase hypothesis, the velocity field is given by a multi-variate Gaussian, whose covariance matrix can be calculated directly from the power spectrum ([16]; [17]; [14]; [48]; [20]; [54]). The velocity correlation function is then a tensor with elements given by:

$$\mathbf{\Psi}_{\mu\nu}(\mathbf{r}) \equiv \langle v_\mu(\mathbf{x}) v_\nu(\mathbf{x}+\mathbf{r})\rangle = \Psi_\perp(r)\delta_{\mu\nu} + \left[\Psi_\parallel(r) - \Psi_\perp(r)\right]\hat{r}_\mu \hat{r}_\nu \quad , \tag{8}$$

where, in linear perturbation theory,

$$\Psi_{\perp,\parallel}(r) = \frac{\Omega^{1.2}}{2\,\pi^2} \int \mathrm{d}k\, P(k) K_{\perp,\parallel}(kR) \quad , \tag{9}$$

and $K_{\perp,\parallel}(x)$ are appropriate combinations of spherical Bessel functions.

Thus, *if measurements of peculiar velocity for different galaxies are independent*, then the covariance matrix between radial peculiar velocities u_i, u_j of two galaxies i and j separated by a distance r is given by:

$$C_{ij} = \hat{\mathbf{r}}_i^\dagger \mathbf{\Psi}(r) \hat{\mathbf{r}}_j + (\Delta u)^2 \delta_{ij} \quad , \tag{10}$$

where the second term on the right-hand side contains the effects of measurement errors. This allows one to write down a simple expression for the likelihood of observing peculiar velocities of a given set of N galaxies, given a power spectrum:

$$\mathcal{L} = [(2\,\pi)^N \det(C)]^{-1/2} \exp\left(-\frac{1}{2}\sum_{i,j}^{N} u_i C_{ij}^{-1} u_j\right) \quad . \tag{11}$$

This has been applied most recently by [54], who used the Mark III peculiar velocity compilation of [51],[52],[53] to constrain the power spectrum (see [25] for an independent determination of the power spectrum from the same data using the statistics of the smoothed $\nabla \cdot \mathbf{v}$). If they do not apply the constraint of the COBE [4] normalization, they find the best-fit CDM models to have a $\Gamma \equiv \Omega h = 0.5 \pm 0.15$, which interestingly calls for *less* large-scale power than has been implied, e.g., by large-scale redshift surveys.

It is not clear, however, whether the error contributions to the covariance matrix (Eq. 10) are purely diagonal. In particular, if there is an error in the assumed distance indicator relation which is used to measure peculiar velocities, or if the distance indicator relation is calibrated from the dataset itself as in [28], covariance is introduced between all peculiar velocities, introducing off-diagonal terms throughout. The effect of this on the determination of the power spectrum remains an area for further work.

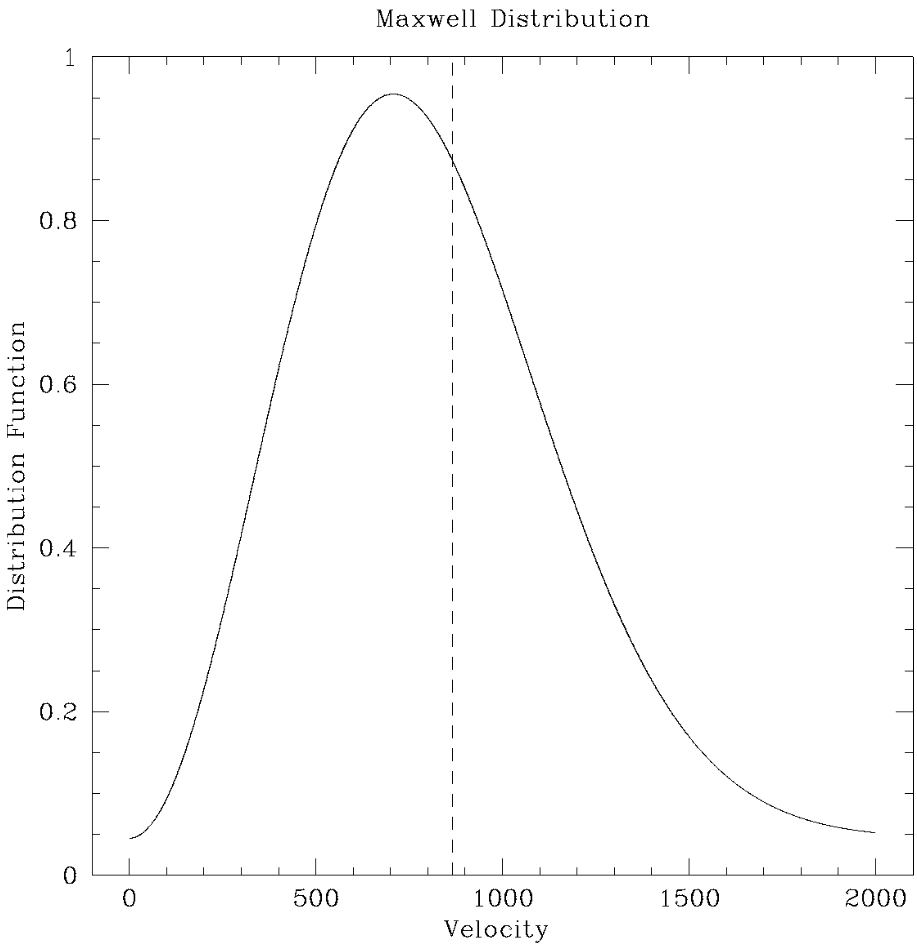

Figure 1: The Maxwellian distribution function of expected bulk flows, on a scale on which the rms value is 866 km s^{-1}. Notice how broad the distribution is.

3 The Predicted Large-Scale Velocity Field: The Observer's View

The observed distribution of galaxies from redshift surveys gives a prediction for the large-scale components of the bulk flow via the integral version of Eq. (6) . In particular, we observe from observations of the dipole anisotropy of the CMB (e.g., [24]) that the Local Group is moving with a velocity of 627 ± 22 km s^{-1} towards $l = 276°$, $b = +30°$ (with 3° errors in each angular coordinate); this is indeed by far the most accurately measured peculiar velocity we know. One can *predict* this peculiar velocity from the observed galaxy distribution to be:

$$\mathbf{v}_{LG} = \frac{\beta}{4\pi n_1} \sum_{\text{galaxies } i} \frac{W(r_i)\,\hat{\mathbf{r}}_i}{\phi(r_i)\,r_i^2} \quad , \tag{12}$$

where $\phi(r)$ is a selection function, to correct for the decrease in density of galaxies as a function of distance in a flux-limited sample, and $W(r)$ is a window function with cutoffs at large and small scales (cf., [46]). The cutoff is needed at large distance because any flux-limited sample has only finite depth, and therefore the dipole one calculates is missing contributions from large scales ([21]; [26]; [32]). Indeed, one might think that the difference between the observed and predicted motion of the Local Group would be a direct measure of large-scale components of the velocity field. Fig. 2 shows the growth of the amplitude and direction of the predicted motion $\mathbf{v}_{LG}(R)$ as a function of the redshift R out to which galaxies are included in the sum, for two redshift surveys: the *IRAS* 1.2 Jy redshift survey ([15]; cf., [46]), and the Optical Redshift Survey ([41]; [42]). Interestingly, the two curves have a very different amplitude, which has interesting things to tell us about the relative bias of *IRAS* and optically-selected galaxies, but discussing that would get us too far afield. For the moment, notice that both curves seem to converge to a constant value (both amplitude and direction) for $cz > 4000$ km s^{-1}, implying that there is little contribution on larger scales. This in turn would imply that the sphere of radius 4000 km s^{-1} is at rest.

Unfortunately, things are not so simple. First, as Juszkiewicz *et al.* [21] pointed out, the difference between the true peculiar velocity and $\mathbf{v}_{LG}(R)$ depends on the position of the center of mass of the sample out to R:

$$\mathbf{v}_{LG}(R = \infty) = \mathbf{v}_{LG}(R) + \mathbf{v}_{bulk}(R) - \frac{1}{3}\beta\, r_{\text{center of mass}} \quad , \tag{13}$$

where $\mathbf{v}_{bulk}(R)$ is the quantity we're interested in in the current context, the mean bulk flow of the sphere out to radius R. One can calculate the rms position of the center of mass of a sample given a power spectrum from linear theory; one finds another integral over the power spectrum like Eq. (7), although with a different smoothing kernel. This term is quite small for small values of R, but becomes comparable to the expected rms bulk flow for values of R above 5000 km s^{-1} or so [44], and indeed, for the *IRAS* 1.2 Jy sample, $r_{\text{center of mass}}$ is of the order of 250 km s^{-1} for an outer radius of 10,000 km s^{-1}.

More important than this, however, are all the additional effects which cause the quantity in Eq. (12) to differ from the theoretical ideal. Non-linear effects, shot noise, assuming the incorrect value of β (which of course we don't know) and the smoothing on small scales all will contribute to the difference between the observed and predicted motion of the Local Group [46]. The most pernicious effect, however, was pointed out by [22]. With a redshift survey, one is measuring the density field in redshift space. However, as Eq. (2) makes clear, this differs from the bulk flow in real space by the effects of peculiar velocities, and to the extent that the peculiar velocity field shows coherence (which of course is what we're trying to get a handle on here), Eq. (13) is systematically biased. In particular, if one's estimate of the velocity of the Local Group itself is off (e.g., if one doesn't correct for the $\mathbf{v}(\mathbf{0})$ term in Eq. (2) at all), the positions of *all* galaxies in the sample are affected in a dipolar way, clearly affecting the predicted motion of the Local Group, and the apparent convergence, or lack thereof, of $\mathbf{v}_{LG}(R)$. Strauss *et al.* [46] find that with their best correction of the density field for peculiar velocities, the *IRAS* dipole indeed seems to converge quite nicely, but even then, there is a very intriguing, large contribution to the dipole (albeit at the 2σ level) between 17,000 and 20,000 km s^{-1}. It will be very interesting to see whether this contribution remains with the just completed PSCZ survey of *IRAS* galaxies to 0.6 Jy (cf., Efstathiou, this volume).

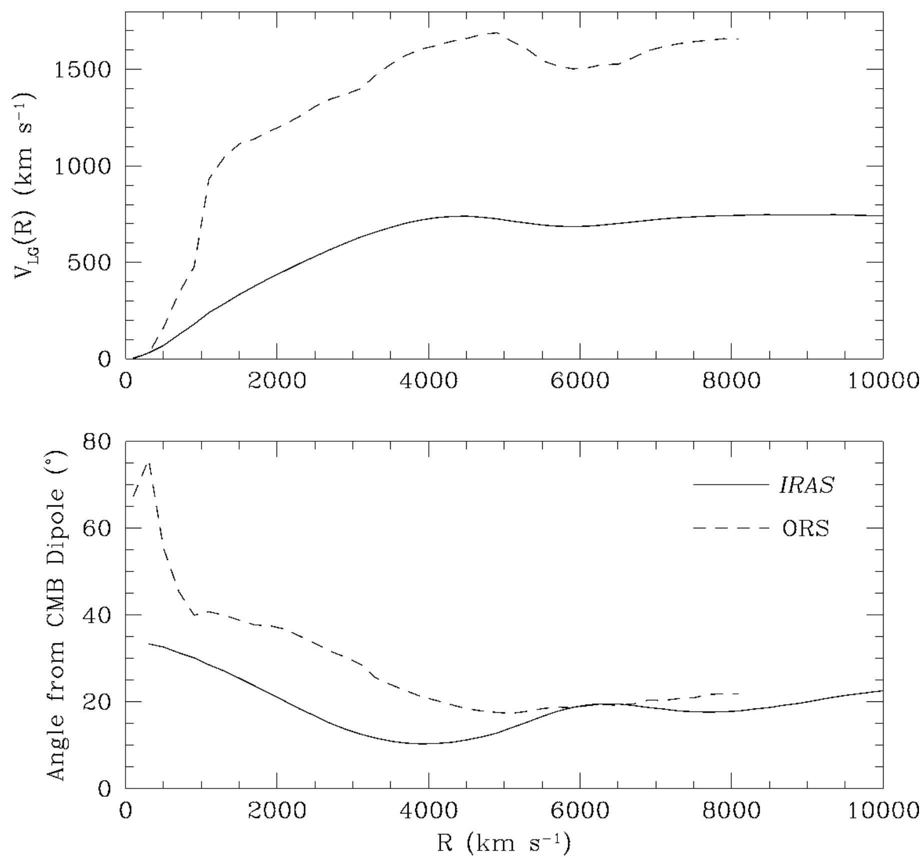

Figure 2: The amplitude (upper panel) and direction relative to the CMB dipole (lower panel) of the gravitational dipole of two surveys, the *IRAS* 1.2 Jy survey (solid lines) and the Optical Redshift Survey (dashed lines). Notice the apparent convergence of the dipole in both cases beyond roughly 4000 km s^{-1} (although the two differ quite a bit in amplitude). Notice also that the ORS is not quite as deep as the *IRAS* sample, and therefore the dipole calculation is cut off sooner.

4 The Measurement of Bulk Flows

The quantity we hoped to get a handle on from the convergence of the density dipole, Eq. (12), is the average peculiar velocity of galaxies within a sphere of radius R centered on the Local Group. One approach is to measure it directly from a full-sky peculiar velocity survey. It is one of the lowest-order statistics one might imagine measuring from a peculiar velocity sample, but it is maximally sensitive to systematic errors in observations between different areas of the sky.

In particular, most peculiar velocity surveys carried out to date have been done over a relatively limited area of the sky. If they are calibrated externally (as they usually are), zero-point differences between the calibrators and the sample will give rise to false bulk flow measurements. Moreover, Malmquist bias can give an artificial signature of outflow [5].

To avoid these problems, we would like to have measurements of peculiar velocities over the full sky. We have already made reference above to the Mark III dataset of [51], [52], [53], which combines one $D_n - \sigma$ [13], and six Tully-Fisher [31]; [30]; [7], [1] (cf., [47]); [50]; [29] peculiar velocity samples. A great deal of work has been done to make these datasets consistent by matching them where they overlap. The bulk flow of the resulting full-sky sample has been calculated by [8], and more recently by Dekel *et al.* (in preparation). I describe the latter calculation here.

The peculiar velocity data are noisy and sample the field sparsely and inhomogeneously. The data can be smoothed if one assumes that the velocity field is derivable from a potential; this allows the calculation of a unique three-dimensional velocity field from observations of radial peculiar velocities (the POTENT method; [12]; [11]; [9]; Dekel, this volume). Calculating the bulk flow is then straightforward, and the results are shown in Fig. 3.

This approach has the advantage that the bulk flow that is calculated is close to the theorist's ideal, the volume-weighted bulk flow. Indeed, the straight fit of individual peculiar velocities in a sample to a bulk flow will not be equivalent to the volume-weighted bulk flow, both because of clustering within the sample (cf., the discussion in [44]) and because of the increasing peculiar velocity errors with distance (cf., [23]).

However, measuring a bulk flow on large scales requires tremendous control over systematic photometric errors. Indeed, a 0.10 mag difference in the photometric zero-points of the Mark III sample from one end of the sky to another would translate into an artificial 300 km s^{-1} bulk flow at 6000 km s^{-1} from the Local Group. Davis *et al.* [10] have carried out a multipole comparison of the Mark III peculiar velocity field with that predicted from the *IRAS* 1.2 Jy redshift survey, and found that there are indeed discrepancies between the two fields beyond 4000 km s^{-1} of roughly 300 km s^{-1} amplitude. It remains unclear whether this is the signature of the gravitational influence of dark matter whose distribution has nothing to do with that of galaxies, a sign that peculiar velocities are not wholly due to the process of gravitational instability, or more prosaically, that there are systematic errors in the Mark III data which are unaccounted for.

In the latter regard, Fig. 3 compares various determinations of the bulk flows of galaxies within spheres centered on us that have been published in the literature. This figure is an updated version of one shown by [36]. Error bars are as given by each author, and do not take into account any misalignment between the error ellipsoids and the the Cartesian axes chosen. The current confused situation is reflected in the large number of non-overlapping error bars in this figure. However, note that the bulk flow within 6000 km s^{-1} of Dekel *et al.* (from the POTENT analysis of the Mark III data) and of [9] (from the POTENT-like analysis of the Giovanelli *et al.* data; cf., Giovanelli in this volume) are in excellent agreement, despite almost completely independent data (they do share the Mathewson *et al.* [29] data in common). It will be very interesting to see if they agree this well shell-by-shell.

5 Full-Sky Peculiar Velocity Surveys

Given the uncertainty introduced by possible zero-point differences between the samples making up the Mark III dataset, how can the bulk flows on large scales best be measured? The ideal way is with a peculiar survey of galaxies covering the entire sky, observed in as uniform a way as possible. In particular, the survey should

- have full-sky, uniform sampling in angle and redshift;
- have well-defined, simple, and easily modeled selection criteria;

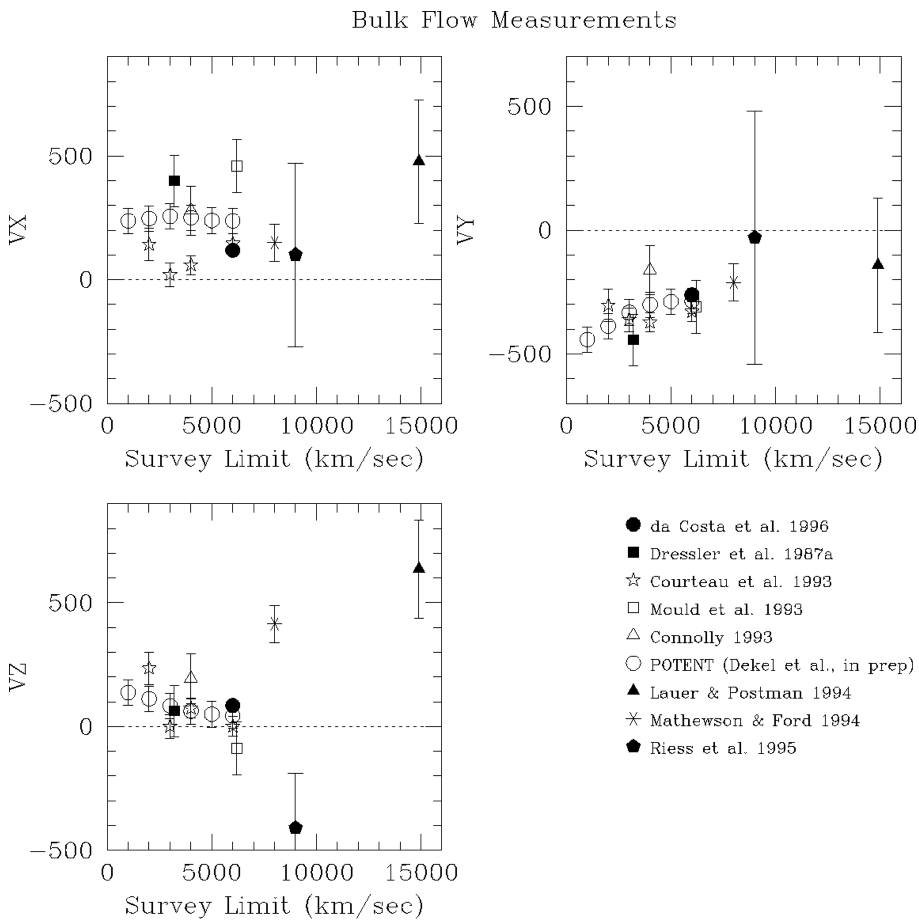

Figure 3: Determinations of the bulk flow of galaxies on various scales from the literature. The three panels give the components of the quoted bulk flows along the Galactic X, Y, and Z directions in km s^{-1}, as a function of the depth of the various surveys. Error bars are as quoted by each paper, and do not take into account the covariance between the different directions (i.e., due to error ellipsoids whose principal axes are not aligned with the Galactic Cartesian directions). Adapted from Postman 1995.

- use a distance indicator with small intrinsic dispersion;
- use uniform observing techniques between North and South, Spring and Fall, with much repeat observations.

There are a number of surveys just completed or in progress now which approach this ideal. In particular:

- Roth [40] and Schlegel [43] have carried out a Tully-Fisher study of a full-sky volume-limited sample of 140 *IRAS* galaxies to 4000 km s^{-1}. A bulk flow analysis has not yet been done, although the data have been compared to the *IRAS* predicted velocity field, and have found consistency for the relatively small value of $\beta = 0.4$ [43].
- The EFAR collaboration [49] has carried out a $D_N - \sigma$ study of over 700 elliptical galaxies in 84 clusters in the Hercules-Corona Borealis, and Perseus-Pisces-Cetus directions in the redshift range $6000 < cz < 15000$ km s^{-1}, with the aim of constraining the velocity fields in these superclusters.
- Hudson *et al.* (in preparation) are extending the EFAR survey with measurements of $D_n - \sigma$ distances to 6-10 ellipticals in those clusters in the Lauer-Postman (1994) sample [28] with redshifts $cz < 12,000$ km s^{-1}.
- Fruchter, Moore & Steidel (in preparation) are doing wide-field photometry of a subsample of the Lauer-Postman clusters; a fit of the photometry to a Schechter function then yields a distance.
- J. Willick has measured Tully-Fisher distances to 20 spirals in each of 15 clusters of galaxies over the sky, at $cz \approx 10,000$ km s^{-1}. Analysis is in progress.
- Giovanelli *et al.* are measuring Tully-Fisher distances to spiral galaxies both in the field and in clusters over a large fraction of the sky to redshifts of 6000 km s^{-1} and greater; see Giovanelli in this volume.

There are two further surveys in which I am involved, which I describe in the following two sections.

6 The Bulk Flow of Brightest Cluster Galaxies

Lauer and Postman [28], [37] presented distances of the Brightest Cluster Galaxies (BCG's) of a sample of 119 Abell [2], [3] clusters to $cz < 15,000$ km s^{-1}. Following work of [18] and [19], they found that the luminosity L of these galaxies within an aperture of radius $10\,h^{-1}$kpc correlated with the logarithmic slope of the surface brightness profile α. This yields a distance indicator with an error of $15-20\%$, depending on the value of α. Their sample was full-sky (or as much so as the zone of avoidance would allow) and volume-limited, and great effort was taken to obtain and reduce the data as uniformly as possible.

To their great surprise, the sample showed a strong signature of bulk flow, with an amplitude of 764 ± 160 km s^{-1} [6], towards $l = 341°$, $b = +49°$. This was much larger than one might expect, given the effective depth of the sample of ≈ 8000 km s^{-1}; indeed, [14] and [44] both showed that a bulk flow with the statistical significance of that of Lauer-Postman ruled out a whole series of cosmological models at the $> 95\%$ confidence level.

As a follow-up to this survey, Tod Lauer, Marc Postman and I are extending the sample to $cz = 24,000$ km s^{-1}. The sample now consists of 529 BCG's, an increase of more than a factor of 4 from the original 119 (the Abell cluster catalog has the beautiful feature of being volume-limited, at least to moderate redshifts, and this increase in number of clusters is almost exactly the increase in volume). The photometry for this sample is all in hand, and redshifts for all BCG's are nearly complete. Barring unseen systematic effects (which we've worked very hard to minimize), we should be able to measure the bulk flow on these scales to $130\,\mathrm{km\ s^{-1}}$ or so. We have also measured velocity dispersions for the BCG's, with preliminary indications that this reduces the scatter in the $L - \alpha$ relation, in analogy to the $D_n - \sigma$ relation. The sky distribution of this sample is shown in Fig. 4a.

As Fig. 3, and the controversy that the Lauer-Postman result have engendered, make clear, the comparison of various measurements of bulk flows with one another is non-trivial.

The velocity field has components on all scales; it is not purely dipolar in nature. The geometry of any given sample couples to various multipoles of the velocity field (the sparser the sampling is, the larger the extent to which

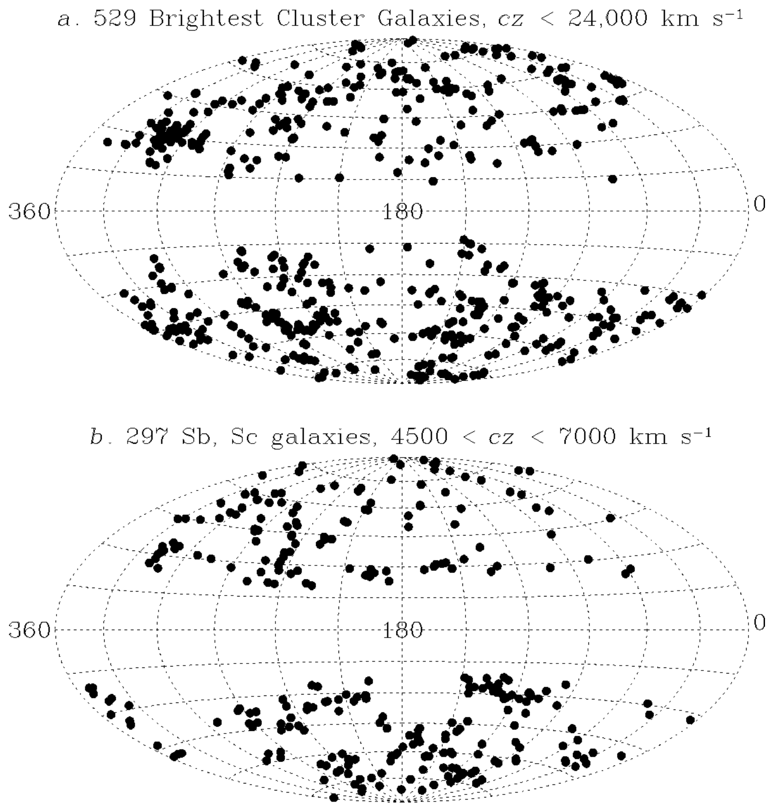

Figure 4: *a.* The BCG sample with $z < 0.08$. The substantial region devoid of clusters in the general direction of the Galactic center is due to the difficulty in finding clusters in regions of high stellar density, and is a general feature of the Abell catalogue. *b.* The sky distribution in Galactic coordinates of galaxies in the Sb, Sc shell sample at $cz \approx 6000$ km s^{-1}.

this is true), and therefore not all bulk flow measurements measure the same quantity [48]. Thus [38] published a bulk flow analysis of 13 Type 1a supernovae, which appear to be standard candles to an accuracy of $\sim 5\%$ [39]. Their results were inconsistent with that found by Lauer & Postman at the 99% confidence level, *assuming that the velocity field was describable by a pure bulk flow plus small-scale incoherent noise.* However, the two surveys sample space really very differently, and therefore are very differently sensitive to components of the velocity field on scales smaller than the dipole. Watkins & Feldman [48] calculated the expectation value of the dot product of the bulk flows each measured, normalized by the expectation value of each bulk flow separately:

$$\mathcal{C} \equiv \frac{\langle \mathbf{U}^{LP} \cdot \mathbf{U}^{RPK} \rangle}{(\langle \mathbf{U}^{LP} \cdot \mathbf{U}^{LP} \rangle \langle \mathbf{U}^{RPK} \cdot \mathbf{U}^{RPK} \rangle)^{1/2}} \quad . \tag{14}$$

This quantity, a sort of dimensionless covariance between the two bulk flow measurements, would be close to unity if these two surveys were indeed measuring the same quantity. The results depend on the power spectrum assumed. If one assumes "realistic" power spectra, the quantity $\mathcal{C}$ is of the order of 10%, but as mentioned above, the Lauer-Postman result is inconsistent with most ordinary power spectra. Watkins & Feldman thus also consider a power spectrum with a huge bump at large scales; in such a model, the relative importance of small-scale components of the velocity field is reduced, but the quantity $\mathcal{C}$ is still only 35%.

7 Resolving the Discrepancies

Wandering through the halls of astronomy departments around the country (or even reading preprints on the astro-ph archive), one hears a lot of interesting statements about the large-scale bulk flow of galaxies within 6000 km s^{-1}:

"The Lauer-Postman result cannot be right; it does not agree with observed bulk flow measurements at 6000 km s^{-1}."

"The Lauer-Postman result cannot be right; it does not agree with the fact that the *IRAS* dipole appears to have converged by 6000 km s^{-1}."

"The observed bulk flow at 6000 km s^{-1} from Mark III is inconsistent with the predictions of the *IRAS* redshift survey."

"The Mark III and da Costa *et al.* [9] dipoles are inconsistent with one another at 6000 km s^{-1}."

Clearly, much of the current controversy centers around the bulk flow at 6000 km s^{-1}. Stéphane Courteau, Marc Postman, Dave Schlegel, Jeff Willick, and I have started a full-sky Tully-Fisher survey of galaxies specifically designed to nail down the bulk flow within a shell centered at 6000 km s^{-1}. We have selected 297 Sb-Sc galaxies with $4500 < cz < 7000$ km s^{-1} with appropriate inclinations and without morphological peculiarities, from the magnitude-limited full-sky redshift survey sample of [42] (we decided against using *IRAS* selection, given the large Tully-Fisher scatter observed for *IRAS* galaxies in [43]). The sky distribution of this sample is shown in Fig. 4b. For each galaxy, we measure the rotation curve using a long slit for the Hα line, and are doing photometry in the V and I bands. We have been granted observing time at Kitt Peak and Cerro Tololo for this survey, and we hope to finish in one year. Our estimate is that we will be able to measure the bulk flow of this shell with an error of 70 km s^{-1}, with an error ellipsoid that will be close to isotropic. We believe that this survey should resolve much of the controversy that is currently swirling around this very hot topic.

I would like to acknowledge my collaborators in the various projects I discuss here: the *IRAS* 1.2 Jy and ORS redshift surveys (Marc Davis, Alan Dressler, Karl Fisher, John Huchra, Ofer Lahav, Basílio Santiago, and Amos Yahil), the POTENT/Mark III analysis (David Burstein, Stéphane Courteau, Avishai Dekel, Sandy Faber, and Jeff Willick), and the two bulk flow projects described above (Stéphane Courteau, Tod Lauer, Marc Postman, David Schlegel, and Jeff Willick). I acknowledge the support of a Fellowship from the Alfred P. Sloan Foundation.

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