

# THE AGE OF THE GALACTIC GLOBULAR CLUSTER SYSTEM

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## ABSTRACT

A careful assessment of current uncertainties in stellar physics (opacities, nuclear reaction rates, equation of state effects, diffusion, rotation, and mass loss), in the chemistry of globular cluster (GC) stars, and in the cluster distance scale, suggests that the most metal-poor (presumably the oldest) of the Galaxy's GCs have ages near 15 Gyr. Ages below 12 Gyr or above 20 Gyr appear to be highly unlikely. If these  $\approx 2\sigma$  limits are increased by  $\sim 1$  Gyr to account for the formation time of the globulars, and if standard Friedmann cosmologies with the cosmological constant set to zero are assumed, then the GC constraint on the present age of the Universe ( $t_0 \geq 13$  Gyr) implies that the Hubble constant  $H_0 \leq 51$  km s<sup>-1</sup> Mpc<sup>-1</sup> if the density parameter  $\Omega = 1$  or  $\leq 62$  km s<sup>-1</sup> Mpc<sup>-1</sup> if  $\Omega = 0.3$ .

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## 1. INTRODUCTION

As fossil relics dating from the formation of the Galaxy and as the oldest objects in the Universe for which reliable ages can be derived, the Galaxy's globular star clusters have been the subject of intensive investigation for more than four decades. Their age distribution and the trends that they define of age with metallicity, position in the Galaxy, and kinematic properties are direct tracers of the chronology of the first epoch of star formation in the Galactic halo. Whether the globular cluster (GC) system encompasses an age range of several billion years or whether the majority of the GCs are nearly coeval is still the subject of lively debate. In a companion review, Stetson, Vandenberg & Bolte (1996) summarize the many advances that have been made in the determination of *relative* GC ages and assess their implications for Galactic formation scenarios. *Absolute* cluster ages—which are the focus of the present study—provide a vital constraint on the age of the Universe and thereby on the cosmological models that are used to describe it. Globular clusters may well have been the first stellar systems to form in the Universe (Peebles & Dicke 1968), probably within approximately  $10^9$  yr after the Big Bang (see Sandage 1993c).

The current widespread interest in securing accurate globular cluster ages results from the dilemma that these ages pose for the presently preferred model in cosmology—a matter-dominated, Einstein-de Sitter universe. This model is characterized by the choice of  $\Omega_{\text{Total}} = 1$  (as required by most formulations of inflation theory), with the cosmological-constant term,  $\Omega_{\Lambda}$ , taken to be zero, implying  $\Omega_{\text{Matter}} = 1$ . In this case, the expansion age of the Universe is given by  $t_0 = (2/3)H_0^{-1}$ , which works out to 8.3 Gyr if the Hubble constant  $H_0$  is taken to be  $80 \text{ km s}^{-1}\text{Mpc}^{-1}$ . Support for this particular value of  $H_0$ , or one within  $\pm 10$ –15% of it, has been boosted by the detection and analysis of Cepheid variables in Virgo cluster galaxies using both the Canada-France-Hawaii Telescope (Pierce et al 1994) and the *Hubble Space Telescope* (Freedman et al 1994; Kennicutt, Freedman & Mould 1995). Moreover, very similar estimates have been favored in most recent reviews of  $H_0$  determinations (e.g. Jacoby et al 1992; Huchra 1992; van den Bergh 1992, 1994).

These results notwithstanding, significant support persists for  $H_0 < 65 \text{ km s}^{-1}\text{Mpc}^{-1}$  (e.g. Saha et al 1994, 1995; Birkinshaw & Hughes 1994; Hamuy et al 1995; Sandage et al 1996); consequently, such lower values cannot yet be ruled out. But, even if  $H_0$  were as low as  $55 \text{ km s}^{-1}\text{Mpc}^{-1}$ , the implied age for the Universe from the standard cosmological model is only 12.2 Gyr,

which is also inconsistent with the GC-based estimate of  $\sim 16$  Gyr.<sup>2</sup> Thus the standard model would appear to fail the “age concordance” test, which is simply that the age of all things in the Universe must be smaller than the elapsed time since the Big Bang. Although there is increasing observational evidence for  $\Omega_{\text{Matter}} \approx 0.3$  (Vogeley et al 1992, Carlberg et al 1996, Squires et al 1996), even for a low-density,  $\Omega_{\Lambda} = 0$  Universe (for which  $t_0 \approx H_0^{-1}$ ), it may not be possible to achieve compatibility with the GC age constraint if  $H_0$  is as high as many people believe. This has led to increasing speculation that the cosmological constant is nonzero (e.g. Efstathiou 1995); however, before stellar age estimates can be used to rule out any cosmologies, a reappraisal of the errors associated with GC age determinations is worthwhile. It is our intent to do just that.

In Section 2 we review the uncertainties in the stellar evolution models due to possible errors in the relevant input physics: nuclear reaction rates, opacities, nonideal gas law effects in the equation of state, and the treatment of convection. In this section we also discuss the effects of input physics that are not normally a component of standard models: rotation, diffusion, and main-sequence mass loss. The observed chemical abundance trends among GC giants are highlighted therein because they provide perhaps the strongest indication of inadequacies in the stellar models for very metal-poor stars. We then briefly consider the possible role of unconventional physics, describe some pertinent observational tests of stellar evolution theory, and briefly recall the very first estimates of GC ages.

As has been recognized for a number of years, the dominant error in the derivation of ages from the luminosity of main-sequence turnoff stars in GCs [which we designate as  $L^{\text{TO}}$ ,  $M_{\text{bol}}(\text{TO})$ , or  $M_V(\text{TO})$ ] is the uncertainty in the Population II distance scale (cf Renzini 1991). In Section 3 we discuss the issue of globular cluster distances. There appears to be a dichotomy developing, with a “long” distance scale based on nearby subdwarfs, the calibration of the horizontal branch (HB) in the LMC, and analyses of the pulsational properties of cluster RR Lyrae variables and a “short” distance scale based on Baade-Wesselink and statistical parallax studies of field RR Lyraes. The disagreement

<sup>2</sup>Since the 1970s there has been general agreement that the oldest of the Galaxy’s GCs has an age somewhere between 14 and 19 Gyr, with  $16 \pm 3$  Gyr being perhaps the most frequently mentioned estimate: See, for instance, Demarque & McClure (1977); Carney (1980); Sandage, Katem & Sandage (1981); VandenBerg (1983); Gratton (1985); Peterson (1987); Buonanno, Corsi & Fusi Pecci (1989); Lee, Demarque & Zinn (1990); Rood (1990); Iben (1991); Renzini (1991); Salaris, Chieffi & Straniero (1993); Sandage (1993c); Chaboyer (1995); Bolte & Hogan (1995); and Mazzitelli, D’Antona & Caloi (1995). These represent a small fraction of the published reviews and original investigations over this period that have reached basically the same conclusion.

in the implied luminosity of the HB from the two calibrations is  $\gtrsim 0.25$  mag. We express some preference for the long distance scale, in which case the implied age for the metal-poor cluster M92 is  $\sim 15$  Gyr: For the short distance scale its age is increased to  $\gtrsim 18$  Gyr. A brief summary of the ramifications of such ages for cosmology is given in Section 4.

## 2. THE STELLAR EVOLUTION CLOCK

Iben & Renzini (1984) and Iben (1991) have written fine reviews of our understanding of the evolution of low-mass stars, and Renzini & Fusi Pecci (1988) have carried out an equally valuable analysis of the degree to which canonical stellar evolutionary sequences satisfy the constraints provided by GC color-magnitude diagrams (CMDs). These papers are well worth reading again: Much of what they contain (which is not repeated here) serves to bolster one's confidence in the adequacy and accuracy of computed stellar models. Although many aspects of the more evolved stages of stars remain problematic, the overall picture of stellar evolution is certainly correct. The main point to stress in this section is that the dependence of the turnoff luminosity,  $L^{\text{TO}}$ , on age—which constitutes the stellar evolution clock—appears to be an especially robust prediction. (The turnoff is defined to be the hottest point along an isochrone, marking the end of the main-sequence stage and the beginning of the subgiant phase.)

### 2.1 *Uncertainty in $L^{\text{TO}}$ Due to Basic Stellar Physics Inputs*

Chaboyer (1995) has recently used the direct approach to ascertain the impact of changes to the basic input physics on GC ages, i.e. he has determined how derived ages would be affected if the nuclear reaction rates, opacities, etc were varied, in turn, by amounts equal to reasonable estimates of their probable errors. Consequently, we use a more indirect means to show, just as Chaboyer has concluded, that present uncertainties in these physical inputs can be expected to have only very minor effects on the ages that are obtained from turnoff luminosities. Our modus operandi reveals, in addition, some of the differences between modern evolutionary calculations and those carried out at earlier times.

**2.1.1 NUCLEAR REACTIONS AND OPACITIES** Figure 1 shows plots of the  $M_{\text{bol}}(\text{TO}) [= 4.72 - 2.5 \log(L^{\text{TO}}/L_{\odot})]$  versus age relationships that have been derived by a number of researchers over the past 25 years. All of the calculations are based on the assumptions that  $Y = 0.20$  and  $Z = 0.0001$  for the mass fraction abundances of helium and the metals, respectively. (Throughout our examination of absolute GC ages, we concentrate on the low-metallicity systems, which are likely to be the oldest.) The locus attributed to Iben (1971)

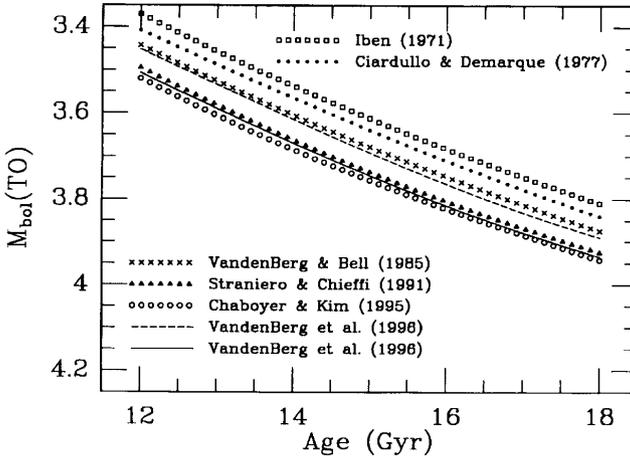


Figure 1 Turnoff luminosity vs age relations from the indicated investigations for the particular choice of  $Y = 0.20$  and  $Z = 0.0001$  for the mass-fraction abundances of helium and the heavier elements, respectively. The  $M_{\text{bol}}(\text{TO})$  values were calculated on the assumption that the solar value is 4.72 mag. Small amounts of smoothing were applied in some instances. The differences between the various loci are discussed in the text.

is based on an analytic expression contained therein, which provides a good approximation to the computations by Iben & Rood (1970) and Simoda & Iben (1970). The latter assume pre-1966 nuclear reaction rates, for the most part, along with the Hubbard & Lampe (1969) set of conductive opacities and the Cox & Stewart (1970) radiative opacity data. Very similar input physics was used in the extensive grid of evolutionary tracks computed by Mengel et al (1979), which were the basis of the Ciardullo & Demarque (1977) isochrones. These were subsequently revised by Green, Demarque & King (1987) to make them better represent real stars. To be specific, the original Yale isochrones were shifted in effective temperature ( $T_{\text{eff}}$ ) to compensate for the fact that Mengel et al tracks were computed for the choice of  $\alpha_{\text{MLT}} = 1.0$  instead of the more realistic value of 1.5. [The quantity  $\alpha_{\text{MLT}}$  is an adjustable parameter in the mixing-length theory (MLT) of convection, which is commonly used in the construction of stellar evolutionary sequences.] Hence both sets of isochrones predict the same dependence of  $M_{\text{bol}}(\text{TO})$  on age (the dotted curve in Figure 1).

VandenBerg & Bell (1985) adopted the updated nuclear reaction rates given by Harris et al (1983) and opacities derived from the Los Alamos Astrophysical Opacity Library (Huebner et al 1977). They noted that revisions to the nuclear physics had a  $\sim 2\%$  effect on the calculated age-luminosity relation for

a given evolutionary track. Because, at low  $Z$ , opacities are completely dominated by the free-free transitions of H and He (cf Schwarzschild 1958), the ramifications of improved determinations of the metal contribution are simply not significant. For instance, the use of the even more modern OPAL opacities (Rogers & Iglesias 1992), which is the main difference between the dashed curve in Figure 1 and the Vandenberg-Bell results, yields essentially the same relation between  $M_{\text{bol}}(\text{TO})$  and age. Indeed, the insensitivity of such relations to the particular generation of opacities assumed gives one considerable confidence in the predictions for, especially, the most metal-poor stars. But, even at higher  $Z$ , turnoff luminosity versus age relations are affected much less by improvements to the opacity than, say, the mass-luminosity relation for zero-age main-sequence stars (e.g. see Figure 4 in Vandenberg & Laskarides 1987). Although enhanced opacities will increase the main-sequence lifetime of a fixed mass, metal-rich star, they will also decrease the turnoff luminosity, such that nearly the same relationship between  $L^{\text{TO}}$  and age is obtained (Rood 1972, Vandenberg 1983). Fortunately, there is good reason for believing that current opacities are uncertain by no more than  $\pm 10\text{--}20\%$ , given that the OPAL data have led to the resolution of several longstanding discrepancies between the predictions of stellar models and actual observations (see the review by Rogers & Iglesias 1994).

**2.1.2 EQUATION OF STATE** The three lowermost curves in Figure 1 differ from the others in one important respect: They allow for Coulomb interactions in the equation of state. Proffitt (1993) was the first to show that this nonideal gas effect causes an  $\approx 4\%$  reduction in age at a given turnoff luminosity for stellar masses and chemical compositions appropriate to the globular clusters. This is close to the difference between the dashed and solid curves, which represent otherwise identical calculations except that the former ignores, and the latter includes, a Coulomb correction term in the free energy. Particularly noteworthy are the Chaboyer & Kim (1995) results: These authors used (in tabular form) the OPAL equation of state (Rogers 1994), which treats several other nonideal effects. They found a 6–7% reduction in age at a given  $M_{\text{bol}}(\text{TO})$ , compared with the case when using the ideal gas law with radiation pressure and electron degeneracy assumed. [Their findings agree well with those of Vandenberg et al (1996) (see Figure 1), whose equation of state was set up to provide a good approximation of the more general OPAL code.]

Judging from the difference (in Figure 1) between the Iben (1971) and the Chaboyer & Kim (1995) results, there has been about a 15% reduction in the predicted age at a fixed  $M_{\text{bol}}(\text{TO})$  over the past 25 years (for the chemical composition that we have been considering). This reduction has resulted from steady refinements in the nuclear reaction rates, opacities, and equation of

state during this time. These aspects of stellar physics are now believed to be sufficiently well understood that future developments in these areas are unlikely to affect predicted ages at more than the few percent level. This conclusion has also been reached by Chaboyer (1995), whose paper contains a useful table giving the fractional age errors as a function of the input physics (also see Renzini 1991).

**2.1.3 CONVECTION THEORY** A more serious concern may be the mixing-length theory of convection. Chaboyer's (1995) calculations show that, although the predicted age-luminosity relation for a given track is not greatly affected by changes in  $\alpha_{\text{MLT}}$ , the age-color (or, equivalently, the luminosity-color) relation is altered in such a way as to shift significantly the luminosity of the hottest point on the track. This is quite an unexpected result. However, while previous studies (e.g. Demarque 1968, Vandenberg 1983) have shown that different assumptions about  $\alpha_{\text{MLT}}$  have profound implications for the temperature scale of an evolutionary track, without affecting its turnoff luminosity, it has apparently not been checked that the turnoffs of isochrones are similarly independent of  $\alpha_{\text{MLT}}$ . In fact, Chaboyer has shown that this is not the case. This is perhaps not too surprising given that the temperature shift induced by a change in  $\alpha_{\text{MLT}}$  is a nonlinear function of mass and evolutionary state (see Figure 3 in Vandenberg 1983).

From a consideration of isochrones computed for values of  $\alpha_{\text{MLT}}$  in the range of 1.0 to 3.0, Chaboyer (1995) has surmised that uncertainties in how to treat convection lead to about a 10% uncertainty in GC ages as inferred from the turnoff luminosity. This is arguably a very generous error estimate given that there is no compelling evidence at the present time to suggest that  $\alpha_{\text{MLT}}$  differs by a large factor between stars of different mass or chemical composition or that it depends sensitively on evolutionary state. Rather, the present observational indications are that the value of  $\alpha_{\text{MLT}}$  needed to produce a realistic solar model is very similar to (possibly even the same as) that needed to explain the lower main-sequence slopes of young open clusters on the CMD (Vandenberg & Bridges 1984), to fit the CMD positions of the local Population II subdwarfs (see Section 3.1 in the present study; Vandenberg 1988), to match the properties of well-observed binaries whose components are in widely separated evolutionary phases (Andersen et al 1988, Fekel 1991), and to reproduce the effective temperatures of GC giant branches as determined by Frogel, Persson & Cohen (1981) from  $V - K$  photometry (Straniero & Chieffi 1991, Vandenberg et al 1996). The last of these is potentially one of the most powerful constraints since the predicted position of the giant branch is highly dependent on the choice of  $\alpha_{\text{MLT}}$  and the comparison between theory and observation is largely independent of the GC distance scale (because the giant branch rises so vertically and

its position varies only slightly with age). Frogel et al (1981) suggest that the uncertainty in their inferred temperatures is  $\pm 90$  K; even if the error were as large as  $\pm 150$  K, this could be accommodated by adopting a value of  $\alpha_{\text{MLT}}$  that differs by as little as  $\pm 0.3$  (see Vandenberg 1983).

The value of  $\alpha_{\text{MLT}}$  cannot be constrained any better than this, given the current observational uncertainties and the sensitivity of model temperatures to many other factors besides convection theory—notably the low-temperature opacities and the treatment of the model atmosphere boundary condition (see Vandenberg 1991). For this reason, the recent suggestion by Chieffi, Straniero & Salaris (1995) that  $\alpha_{\text{MLT}}$  appears to be a weak function of metallicity is not convincing. Their fits to GC giant branches on the  $M_{\text{bol}}-\log T_{\text{eff}}$  plane required a value of  $\alpha_{\text{MLT}} = 1.91 \pm 0.05$  for clusters of intermediate metal abundance ( $Z \approx 0.001$ ), whereas the slightly smaller value,  $1.75 \pm 0.1$ , was needed for the globulars having  $Z \approx 0.0001$ . Obviously, such a small variation is well within the noise of its determination. One is instead impressed (once again) by the fact that stellar models are able to reproduce the observed properties of very different stars with little (or no) variation in  $\alpha_{\text{MLT}}$ . It would be an astonishing result if  $\alpha_{\text{MLT}}$  were constant, because there is no reason whatsoever why it should be; however, what variation there is in this parameter appears to be quite small.

The Chaboyer (1995) investigation does, however, raise the specter that a more realistic theory for convection than the MLT may have significant ramifications for GC ages. This possibility has been given considerable impetus by Mazzitelli, D'Antona & Caloi (1995), who have found that the predicted ages of the most metal-poor GCs are reduced by  $\sim 2$  Gyr simply as a consequence of replacing the MLT by the Canuto & Mazzitelli (1991, 1992; hereafter CM) theory of turbulent convection. This theory, unlike the MLT, allows for a full spectrum of turbulent eddies, and it has essentially no free parameters: The mixing length is taken, at any point in the convective envelope, to be the geometrical depth from the upper boundary of the convection zone. [This choice for the scale length is claimed to be reasonable on the grounds of physical analogies (e.g. with the Earth's atmosphere) and its consistency with the physical scale length at which the superadiabatic zones inside stars grow and fade. Indeed, from the observed  $p$ -mode solar oscillation frequencies, Basu & Antia (1994) have found that envelope models based on the CM formalism provide a much closer match to the inferred structure of the Sun's convection zone than those constructed assuming the MLT. More recent developments (see Rosenthal et al 1995), however, suggest that this agreement may not necessarily imply such a clear-cut preference for the CM theory over the MLT.]

Mazzitelli et al (1995) find that  $T_{\text{eff}}$  effects alone lead to an apparent decrease in the turnoff luminosities of CM isochrones relative to those obtained using

the MLT. (A representative example of their results is illustrated in Figure 2 for an assumed metallicity  $Z = 10^{-4}$ .) They verified that the temporal variation of luminosity and central hydrogen abundance along an evolutionary track is independent of how surface convection is treated (as it should be), concluding that it is the differences in morphology of the CM tracks that give rise to the decrease in  $M_{\text{bol}}(\text{TO})$  in the corresponding isochrones, compared with MLT predictions. However, an age estimate based strictly on the turnoff luminosity will necessarily have a large uncertainty (in addition to those arising from, e.g. distance or chemical composition errors) because of the inherent difficulty in determining that point. By definition, an observed color-magnitude diagram is vertical at the turnoff; consequently, random photometric scatter or small systematic errors in the color calibration can easily cause the estimated magnitude of the bluest point to be in error by 0.1 mag (if not more)—thereby changing the derived age by at least 10%. But this uncertainty can be significantly reduced if, once the cluster distance is set using one or more standard candles (see Section 3), theoretical isochrones for the applicable chemical abundances are shifted horizontally (i.e. in color) by whatever amount is necessary to obtain a best-fit to the main-sequence photometry, and then the age is inferred from the coincidence of the predicted and observed subgiant-branch loci. The level of the subgiant branch (say, midway between the turnoff and the base of the RGB) is clearly a much better luminosity diagnostic than the turnoff point and, moreover, it is insensitive to the choice of convection theory (see Figure 2). (Granted, in the case of models that employ the MLT, large variations in  $\alpha_{\text{MLT}}$

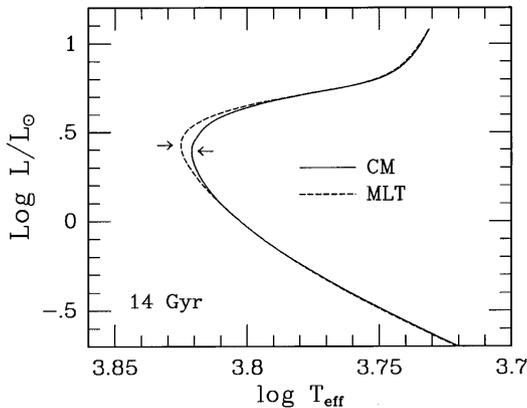


Figure 2 Comparison of Mazzitelli et al (1995) isochrones for  $Z = 0.0001$ , an age of 14 Gyr, and two different treatments of convection (see text). To obtain a superposition of the main-sequence loci, the MLT predictions were shifted by  $-0.0025$  in  $\log T_{\text{eff}}$ . Arrows indicate the turnoff points.

would have some impact on the location/shape of the subgiant branch, but the value of this parameter appears to be fairly well constrained.) The treatment of convection need not, therefore, be a serious concern for the determination of GC ages.

## 2.2 *Uncertainty in $L^{\text{TO}}$ Due to Additional Physics Usually Ignored*

There are (at least) three additional physical processes that can potentially influence the estimated ages of globular cluster stars: 1. atomic diffusion (or gravitational settling), 2. rotation, and 3. mass loss.

2.2.1 ATOMIC DIFFUSION Noerdlinger & Arigo (1980) were the first to construct models for low-mass, Population II stars in which helium was allowed to settle under the influence of gravity and thermal diffusion. They found that He diffusion tends to speed up a star's main-sequence evolution, with the result that the evolutionary tracks had slightly lower turnoff luminosities and effective temperatures compared with their nondiffusive counterparts. This translated into about a 22% reduction in the estimated ages of the globulars if the turnoff luminosity were used as the sole criterion for determining age. The follow-up study by Stringfellow et al (1983) added the interesting result that, as stars ascend the red-giant branch (RGB), the remixing of the outer layers by the deepening envelope convection erases much of the evidence of the settling of helium, and the tracks with and without diffusion gradually converge.

Nearly a decade later, Proffitt & Michaud (1991) computed a new set of diffusive models for metal-poor dwarfs using the improvements to the input physics that had occurred in the meantime—mainly to the diffusion coefficients (Paquette et al 1986). The turnoff luminosities of these models appeared to be significantly less affected by diffusion than the earlier calculations had predicted. And, in fact, the isochrones computed shortly thereafter by Proffitt & Vandenberg (1991) and by Chaboyer, Sarajedini & Demarque (1992) revealed that the age at a given  $L^{\text{TO}}$  is reduced by  $\lesssim 10\%$  due to the gravitational settling of helium. [These investigations also suggested that the impact of diffusion on cluster ages would be appreciably less than this if the latter were obtained from a calibration of the magnitude difference between the horizontal branch and the turnoff. Somewhat reduced HB luminosities, compensating for  $\gtrsim 1/2$  of the decrease in  $L^{\text{TO}}$ , is the expected consequence of differences in the envelope helium contents in the precursor red giants: Not all of the helium that had previously settled below the surface convection zone is dredged back up when the convection attains its deepest penetration on the lower RGB. Hence the envelopes of stars in more advanced evolutionary stages will be characterized by lower  $Y$ , which has the stated effect on HB luminosities (see, e.g. Sweigart & Gross 1976).]

However, atomic diffusion is not without its difficulties. As shown by, e.g. Michaud, Fontaine & Beaudet (1984) and Chaboyer & Demarque (1994), stellar models that allow for diffusion appear to be unable to explain the lithium abundance plateau (Spite & Spite 1982), which is the near constancy of Li abundance in halo stars having  $T_{\text{eff}} > 5500$  K (see, as well, Thorburn 1994). They also yield isochrones that are morphologically distinct from observed globular cluster CMDs (Proffitt & Vandenberg 1991). In contrast, the same investigations show that such data can be matched extremely well by standard, nondiffusive calculations. (The shapes of isochrones are altered by diffusion because it causes a rapid settling of helium in the very metal-poor stars, in particular, from the thin surface convection zones that they possess during their main-sequence phases. This leads to reduced turnoff temperatures by 200–300 K, whereas, as already mentioned, giant-branch effective temperatures remain relatively unaffected. One must always be wary of drawing strong conclusions from  $T_{\text{eff}}$ /color comparisons, but it seems unlikely that current estimates of the temperatures of turnoff stars are uncertain by much more than  $\pm 100$  K.)

Why, then, is diffusion so problematic for Population II stars when it is not for, e.g. the Sun? Indeed, helioseismic data indicate a clear preference for solar models that include its effects (see Guzik & Cox 1992, 1993; Christensen-Dalsgaard, Proffitt & Thompson 1993). Because diffusion is such a fundamental physical process, which should occur in all stars, one can only conclude that something must be inhibiting its importance in metal-deficient stars. Suggested possibilities include turbulence (Proffitt & Michaud 1991), rotation (Chaboyer & Demarque 1994), and mass loss at the level of  $\approx 10^{-12} M_{\odot} \text{ yr}^{-1}$  (Swenson 1995). Turbulent mixing below the surface convection zone will slow the rate at which the surface He abundance decreases, but as demonstrated by Proffitt & Michaud (1991), it cannot eliminate the gravitational settling of helium without destroying more lithium than is consistent with the Spite plateau. The combined rotation-diffusion models of Chaboyer & Demarque (1994) are able to match the Li observations reasonably well, but they predict essentially the same evolutionary tracks on the H-R diagram as the pure diffusion calculations; consequently, the  $T_{\text{eff}}$  scale problems noted above would remain. Finally, Swenson's (1995) work has revealed that stellar models that treat diffusion can be made compatible with the Li data, and possibly even with globular cluster CMDs, if mass loss is assumed to occur at modest rates during main-sequence evolution.

Although stellar models that include these additional processes, which must operate to some extent in real stars, do not satisfy the observational constraints quite as well as one would hope, they do go a considerable distance towards overcoming the initial objections to diffusion. It is entirely possible that improved treatments of turbulence, rotation, and/or mass loss will reduce the remaining

discrepancies concerning the surface properties of Population II stars, but they would not affect the shortening of main-sequence lifetimes due to He diffusing into the stellar cores (unless rapid core rotation could do so). Accordingly, in our view, there is really very little basis for ignoring the implications of diffusion for GC ages, which amount to less than a 10% reduction in age at a given turnoff luminosity. [This value may be slightly revised when models for GC stars become available that allow for the settling of heavier elements such as C and Fe, whose diffusion velocities are comparable with that of helium (Michaud et al 1984). Proffitt's (1994) latest solar models indicate that heavy-element settling causes only minor structural changes beyond those arising from He diffusion alone.]

**2.2.2 ROTATION** Rotation clearly has considerable potential in its own right to alter stellar ages (cf Law 1981), and given the abundance of direct and circumstantial evidence for rotation in GC stars, one might surmise that it has a significant impact on the ages of these objects. From the broadened lines evident in echelle spectra, Peterson (1985a,b) determined that the blue HB stars in a number of GCs rotate at significant rates (typical  $v \sin i$  values of  $\sim 10$ – $20 \text{ km s}^{-1}$ ). Moreover, she found that the mean rotation speeds were directly correlated with the ratio  $B/(B+R)$ , where  $B$  represents the number of HB stars to the blue of the instability strip and  $R$  denotes the number to the red. That is, the bluer a cluster's horizontal branch, the faster its stars rotate, on average. A qualitatively similar correlation exists between the apparent cluster ellipticity and HB type (Norris 1983): None of the most highly flattened globulars has a red horizontal branch. Although the ellipticity is presumably a reflection of the total cluster angular momentum, the Peterson data suggest that it may also be indicative of the amount of rotational angular momentum contained within individual member stars. Nonetheless, the Peterson observations do provide some support for those suggestions (e.g. by Fusi Pecci & Renzini 1975, Renzini 1977) that rotation could be a significant factor in determining the morphology of the horizontal branch. Importantly, they also show that at least some GC stars are able to retain substantial amounts of angular momentum in their interiors throughout their evolutionary histories.

Another possible signature of rotation is the observed spread in color/ $T_{\text{eff}}$  encompassed by a globular cluster's HB population. Such data seem to require that there is a large variation in mass among the core He-burning stars and that their mean mass is significantly smaller than that of stars presently near the turnoff (cf Rood 1973). Variable amounts of mass loss, driven (perhaps) by star-to-star differences in rotation rate, must therefore occur either during the giant-branch evolution and/or as a consequence of the helium flash event itself.

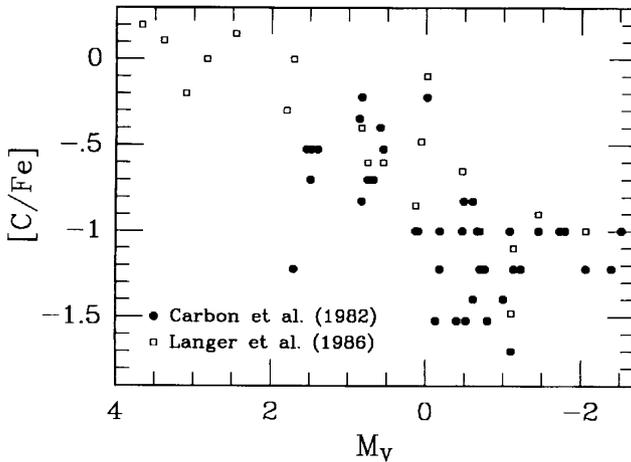


Figure 3 Carbon abundance as a function of  $M_V$  in M92. Only those data not flagged as being uncertain in the indicated studies, from which the observations were obtained, have been plotted.

The huge variations in the measured strengths of the CN, CH, and NH bands, and in the inferred or derived abundances of C, N, and O among bright GC giants (see the review by Kraft 1994 and references therein) provide further indirect evidence for the presence of rotation. These variations are not predicted by canonical evolutionary theory, but are plausibly explained in terms of circulation currents spawned by rotation (Sweigart & Mengel 1979, Smith & Tout 1992); or they may arise as the result of thermal instabilities in the H-burning shell (see Schwarzschild & Härm 1965; Von Rudloff, VandenBerg & Hartwick 1988). Especially compelling are those observations that show a dependence of molecular band strengths on giant-branch luminosity. For instance, as shown in Figure 3, the mean C abundance in M92 stars appears to decline continuously with advancing evolutionary state: Very similar trends have been observed in M15 (Trefzger et al 1983) and in NGC 6397 (Briley et al 1990). The data that have been plotted exhibit a spread of up to  $\sim 0.7$  dex in  $[C/Fe]$  at a given  $M_V$ , which could be accounted for if some stars mix more than others due to differences in their angular velocities. ( $[C/Fe]$  represents the logarithm of the C/Fe number abundance ratio in an observed star minus the logarithm of the same quantity in the Sun, e.g.  $[C/Fe] = -1$  means that the measured carbon-to-iron ratio is one tenth of the solar value.) Low (or super-low) oxygen abundances are also being found in the same GCs—see Pilachowski (1988), Sneden et al (1991), and Bell, Briley & Norris (1992) regarding M92, M15 and M92, and NGC 6397, respectively—as well as in M13 (Brown, Wallerstein & Oke 1991;

Kraft et al 1993). The critical point is that nitrogen tends to be anticorrelated with C and O, often (though probably not always) to the extent that  $C+N+O$  is a constant (see Pilachowski 1989). This strongly suggests that the envelopes of bright giants in many globular clusters are somehow connected to the nuclear burning shell regions and are experiencing ongoing  $C \rightarrow N$  and  $O \rightarrow N$  processing. Indeed, ad hoc simulations that extend and maintain convective mixing down to the vicinity of the H-burning shell appear to be able to reproduce the observed abundance trends (see Vandenberg & Smith 1988).

The hypothesis that rotationally induced deep mixing is responsible for “anomalous” abundances in GC giants has become more credible during the past couple of years. Such mixing might also explain the correlations between, e.g. the strengths of sodium and aluminum lines with that of the CN band (Peterson 1980), which have been particularly difficult to fathom. It was generally supposed (cf Cottrell & Da Costa 1981) that such data indicated the existence of primordial abundance fluctuations, which is to say that the gas out of which the cluster stars formed was not well-mixed chemically. But a separate explanation for these anomalies may not be needed. Furthering the work of Denisenkov & Denisenkova (1990), who first explored the possibility that  $^{22}\text{Ne}(p,\gamma)^{23}\text{Na}$  operated at the same temperatures as  $O \rightarrow N$  burning, Langer, Hoffman & Sneden (1993) showed that deep mixing to this region of a star would naturally produce an N-Na correlation. In addition, they pointed out that the rate of  $^{25}\text{Mg}(p,\gamma)^{26}\text{Al}$  was essentially the same as that of the aforementioned reaction and hence that, as long as  $^{22}\text{Ne}$  and  $^{25}\text{Mg}$  were present, the production of nitrogen by  $O \rightarrow N$  cycling would be accompanied by the production of  $^{23}\text{Na}$  and  $^{26}\text{Al}$ . Because this occurred at a somewhat cooler temperature than the location of the H-burning shell, mixing into this region would not supply additional fuel into the hydrogen shell and would not, therefore, alter stellar evolution lifetimes.

Thus, a reasonably satisfactory explanation could be offered for the many observations revealing large overabundances of Na and Al, N-Na and N-Al correlations, and O-Na and O-Al anticorrelations (e.g. Cohen 1978; Norris & Freeman 1983; Paltoglou & Norris 1989; Pilachowski 1989; Lehnert, Bell & Cohen 1991; Drake, Smith & Suntzeff 1992; Kraft et al 1992). The main point of disagreement concerned the Langer et al (1993) prediction that the range in Al abundances should be  $\Delta[\text{Al}/\text{Fe}] \lesssim 0.3$ , whereas the observed variation can be as high as  $\Delta[\text{Al}/\text{Fe}] \sim 1.2$  (also see Norris & Da Costa 1995). But with even deeper mixing, and with large initial abundances of the  $^{25,26}\text{Mg}$  seed nuclei [the  $^{26}\text{Mg}(p,\gamma)^{27}\text{Al}$  reaction can operate at a significant rate just below the  $O \rightarrow N$  shell], these data can also be matched by proton-capture nucleosynthesis models (Langer & Hoffman 1995). These models predict that large aluminum

enhancements should be accompanied by observable ( $\sim 0.2$  dex) depletions in Mg (initially mostly  $^{24}\text{Mg}$ , which does not burn). These depletions may already have been detected: Smaller magnesium abundances appear to distinguish the super-oxygen-poor stars in M13 from those having higher oxygen abundances (MD Shetrone, private communication 1995).

An important consequence of the extra-deep mixing, according to Langer & Hoffman (1995), is that significant depletions of the envelope H abundance (or, equivalently, enhancements in the surface helium abundance) amounting to  $\sim 3$ –10% or more would likely occur. This would have some implications for the upper-RGB lifetimes of the affected stars, and it would influence their later evolution. As is well known (cf Rood 1973), higher envelope-helium contents make for hotter and somewhat brighter core He-burning stars. Langer & Hoffman suggest that this may help to explain why M13 has such a blue HB: The bluest stars could be characterized by higher  $Y_s$  in their envelopes. (Alternatively, or in addition to this, these stars could have been subject to especially severe mass-loss rates.) Curiously, Moehler, Heber & de Boer (1995) find that high helium abundances seem to be necessary to explain the spectra of extremely blue HB stars in M15.

Not surprisingly, there are some concerns. For one, the meridional circulation mechanism should not work prior to the H-burning shell contacting the chemical composition discontinuity that is produced when the envelope convection attains its greatest penetration (near the base of the RGB): The significant mean molecular weight gradient between the energy-producing shell and that discontinuity (see Sweigart & Mengel 1979) should inhibit circulation (Tassoul & Tassoul 1984) until contact between the two is made. This contact should occur near  $M_V = 0$  in very metal-poor stars (VandenBerg 1992), at which point there should be a brief hesitation in the rate of evolution up the giant branch as the H-burning shell adjusts to a higher hydrogen abundance (also see Iben 1968a, Sweigart & Gross 1978). In fact, Fusi Pecci et al (1990) claim to have detected the consequent bump in the RGB luminosity function at very close to this magnitude. Yet the progressive depletion of carbon in, e.g. M92 apparently begins fainter than  $M_V = 3$  (see Figure 3), which would seem to be totally at odds with the theory and the Fusi Pecci et al observation.

This dilemma has yet to be resolved. On the one hand, the RGB bump is a very subtle feature in metal-deficient clusters; indeed, Fusi Pecci et al (1990) had to coadd the data for several of these objects in order to improve the signal-to-noise ratio. On the other hand, the measured C abundances are the least secure for the faintest stars, and perhaps these data are in need of significant revision. One point in favor of the abundance data, though, is that the subgiant and giant branch luminosity functions of  $[\text{Fe}/\text{H}] \sim -2$  GCs show anomalous

features and cannot be adequately matched by standard evolutionary predictions (see Bergbusch 1990, Stetson 1991, Bolte 1994). This point is discussed further in Section 2.5. Clearly, further and better observations are required.

Another concern is that deep mixing cannot be invoked to explain all of the abundance data. In M92, for instance, not all stars with low C abundances are nitrogen rich (Carbon et al 1982). Furthermore, despite the evidence for progressively lower  $^{12}\text{C}/^{13}\text{C}$  ratios with increasing luminosity in some GCs (Smith & Suntzeff 1989), low carbon isotope ratios are often found in CN-normal stars, which have presumably not undergone substantial mixing (Bell, Briley & Smith 1990). Moreover, many clusters show bimodal distributions of CN-band strengths (Smith & Norris 1982; also see the review by Smith 1987), which persist right down to the main-sequence turnoffs in some globular clusters (e.g. 47 Tucanae; see Briley, Hesser & Bell 1991). The all-important point here is that the ratio of CN-strong to CN-weak stars does not appear to change with evolutionary state (Smith & Penny 1989, Smith & Norris 1993, Briley et al 1994). The existence of some level of primordial abundance variations would seem to be the inescapable conclusion.

Several general results bring home the complexity of globular cluster abundance work. First, as already implied, RGB mixing tends to become less severe as the cluster metallicity increases (Bell & Dickens 1980, Briley et al 1992). In metal-poor systems, CNO abundances seem to vary with  $M_V$  along much of the giant branch, whereas CN bimodalities with little or no dependence on evolutionary state appear to be characteristic of the more metal-rich globulars. Are we to conclude from this that the mean rotation rates of stars in GCs vary in some systematic way with [Fe/H]? If so, does this impact on derived ages and the age-metallicity relation that describes these objects? Second, even at the same metal abundance, different clusters show a considerable variation in their observed chemistry. Adding to the Suntzeff (1981) study of M3 and M13, which have essentially identical [Fe/H] values, Kraft et al (1992) report that they have been unable to find any super-low-oxygen stars in M3, whereas they comprise  $\sim 15\%$  of the brightest giants in M13. Furthermore, although M13 stars tend to have low oxygen and high sodium abundances, those in M3 are O rich and Na poor. Do M13 stars rotate more rapidly than those in M3 and is this the reason why the two clusters also exhibit very different horizontal-branch morphologies? And finally, Population II field giants do not show the extreme abundance patterns seen in GC giants. No CN-strong stars are found in the field (Langer, Suntzeff & Kraft 1992), and many have noted the lack of field stars with high [Na/Fe] or [Al/Fe] (e.g. Brown & Wallerstein 1993, Norris & Da Costa 1995). Is it, then, a risky procedure to use field RR Lyraes to determine cluster distances? For that matter, how safe is it to use canonical horizontal-

branch models to set the globular cluster distance scale given that the precursor RGB stage is problematic?

It is hard to deny the importance of rotation in GC stars and how it affects their evolution. However, precisely defining and quantifying the role that it plays are not easily accomplished in view of the relatively crude understanding that we presently have of turbulence, circulation, and angular momentum transport in rotating stars (see, e.g. Zahn 1992). Still, exciting progress is being made through such studies as those by Wasserburg, Boothroyd & Sackmann (1995) and Charbonnel (1995), who have been able to account for, among other things, the low  $^{12}\text{C}/^{13}\text{C}$  ratios in bright giants by invoking meridional circulation. Further investigations along these lines are strongly encouraged.

But are the rotation rates of stars in clusters sufficient to significantly affect the relation between age and  $L^{\text{TO}}$  predicted by standard, nonrotating stellar models? The answer to this question is “probably not.” Deliyannis, Demarque & Pinsonneault (1989) have computed a number of evolutionary sequences for low-mass, low-metallicity stars in which internal rotation is followed using the moderately sophisticated code described by Pinsonneault et al (1989). The transport of angular momentum due to rotationally induced instabilities, the angular momentum loss due to a magnetic wind, and the effects of rotation on the chemical abundance profiles are all calculated. In addition, the various free parameters in the theory have been constrained to satisfy the global properties of the Sun, including its present rotation rate and oblateness. Very encouraging is the fact that, as noted by Pinsonneault et al, the predicted rotation in the solar interior is in qualitative agreement with the estimates from oscillation data, especially at radii  $> 0.6R_{\odot}$ . The code has proven successful in modeling the observed surface Li abundances and rotation rates of stars in young open clusters (Pinsonneault, Kawaler & Demarque 1990) and in the halo (Pinsonneault, Deliyannis & Demarque 1992).

The calculations of Deliyannis et al (1989) for globular cluster parameters predict that rotation will not change the age at a given turnoff luminosity by more than 1%. Moreover, they suggest that, due to angular momentum redistribution and losses at the stellar surface, the angular momentum of the core is kept at a level that is insufficient to alter canonical estimates of the core mass at the helium flash. As a result, HB luminosities should not be affected nor should calibrations of the age dependence of the magnitude difference between the turnoff and the horizontal branch. At the same time, the models are expected to possess sufficient differential rotation with depth (according to Pinsonneault, Deliyannis & Demarque 1991), to be capable of matching the rotational velocity data that Peterson (1985a,b) has obtained for the HB stars in several GCs. (These inferences are based on the rotational characteristics of turnoff models: As far

as we are aware, the tracks have not yet been extended past the lower RGB.) Worth repeating is the comment by Pinsonneault et al that “the thin surface convection zones of halo stars allow differential rotation with depth to begin much further out and to reach a greater contrast between central and surface rotation.” This offers the reason why one might expect rotation (and mixing?) to become more important as the cluster metallicity decreases.

All of this represents a really superb theoretical effort and further progress is eagerly anticipated. In particular, it will be interesting to learn whether or not these models can explain the wealth of chemical abundance data previously described. Until those constraints are matched, we suspect that it is still within the realm of possibility that rotation has a bigger effect on turnoff ages than Deliyannis et al (1989) have estimated. However, extremely high rotation rates can be precluded simply because the turnoff stars in GCs follow very tight color-magnitude relationships—see, e.g. Stetson’s (1993) review, wherein he reports that M92’s photometric sequence is only 0.0078 mag thick in  $B - V$  in the range  $17.8 < V < 18.4$  (which is just above the turnoff). If the stars in clusters had high rotation rates, then much larger intrinsic spreads would be expected because of star-to-star differences in rotational velocity and the dependence of a star’s photometry on the particular aspect being viewed (e.g. see Faulkner, Roxburgh & Strittmatter 1968). On the other hand, judging from the very simplistic models by Mengel & Gross (1976), who treat rotation in the spherically symmetric approximation, fairly large rotation rates are needed to cause significant departures from the evolutionary track that a nonrotating star of the same mass and chemical composition would follow. That is, the tightness of observed CMDs does not preclude rotation rates that are large enough to have small effects on the core mass at the helium flash or to change computed age versus turnoff luminosity relations at the few percent level (see the Mengel & Gross study). Note that rotation tends to increase the age at a given  $L^{TO}$ .

**2.2.3 MASS LOSS** A few years ago, Willson, Bowen & Struck-Marcell (1987) suggested that significant mass loss ( $\gtrsim 10^{-9} M_{\odot} \text{ yr}^{-1}$ ) may occur in the region of the main sequence that overlaps with the extension of the Cepheid instability strip. This mass loss would be driven by pulsation as well as the rapid rotation normally possessed by the early-A to mid-F stars that occupy this region. They suggested, for instance, that the Sun’s very low Li abundance (Steenbock & Holweger 1984) could be explained if it started out as a  $2 M_{\odot}$  star and lost half its initial mass during the first  $10^9$  yr of its existence as it evolved through this critical zone on the H-R diagram. According to Willson et al, this mechanism might also account for blue stragglers, and it may even help to alleviate the apparent conflict between GC ages and the age of the Universe implied by high values of  $H_0$  (should they prove to be correct).

Noting that the instability strip crosses the main sequence very near to where the so-called Li dip<sup>3</sup> occurs in Population I stars of type F, Schramm, Steigman & Dearborn (1990) considered whether or not the Willson et al (1987) hypothesis might also work here. They found that models that lose mass at  $\gtrsim 7 \times 10^{-11} M_{\odot} \text{ yr}^{-1}$  are able to match the shape of the Li dip in the Hyades quite well. However, the mass-loss rate had to be  $< 1 \times 10^{-10} M_{\odot} \text{ yr}^{-1}$  in order to avoid being in conflict with the observation that beryllium is not depleted (Boesgaard & Budge 1989). In their much more extensive study, Swenson & Faulkner (1992) agreed that mass-loss models are capable of matching the Li contents of Hyades F stars, but that very little leeway is allowed in the mass-loss rates, which must vary nonmonotonically with initial stellar mass in a very well-defined way, with little star-to-star deviation. [Note that there are alternative explanations for all or part of the Li-dip observations, including atomic diffusion (Michaud 1986) and rotationally induced mixing (e.g. Charbonnel & Vauclair 1992).]

Based on the existence of a few extremely metal-deficient stars with very low Li abundances, Dearborn, Schramm & Hobbs (1992) suggested that an analogous lithium dip might be present on the Population II main sequence. Further, they found that such data could be explained by the same mass-loss model as was used for the Hyades if mass-loss rates of  $\sim 10^{-11} M_{\odot} \text{ yr}^{-1}$  were assumed to apply within an instability strip lying in the range  $6600 \leq T_{\text{eff}} \leq 6900$ . They commented that mass loss of this type would make GCs look  $\sim 1$  Gyr older than they really are (also see Shi 1995). However, Molaro & Pasquini (1994) have detected lithium in a turnoff star of the  $[\text{Fe}/\text{H}] \approx -2.1$  globular cluster NGC 6397, and, moreover, their measured Li abundance is the same as those of field halo stars (e.g. Thorburn 1994) to within the errors. This observation provides a very strong argument against the high mass-loss hypothesis, especially given that the field star data themselves preclude mass-loss rates  $\gtrsim 2 \times 10^{-12} M_{\odot} \text{ yr}^{-1}$  (Swenson 1995). (It also gives a reassuring indication of the similarity between cluster and field main-sequence stars at low  $Z$ .)

As discussed by Shi (1995), there is another way to test whether or not the turnoff stars in GCs are losing significant amounts of mass. If they are, then a step-like feature, reflecting the sudden onset of high mass-loss rates, should manifest itself in the luminosity function plane at the point where the CMD intersects the instability strip. Although there are some anomalous features in the observed luminosity functions for GCs, mainly for the most metal-deficient systems (see Section 2.5), they appear to be restricted to post-turnoff evolutionary phases. That is, no obvious bumps or steps are seen at turnoff luminosities.

<sup>3</sup>The Li dip refers to the striking variation of Li abundance with  $T_{\text{eff}}$  that Boesgaard & Tripicco (1986) discovered in the Hyades: Stars with  $T_{\text{eff}}$ 's near 6600 K show severe Li depletions compared with those 300 K cooler or hotter.

All in all, the possibility that mass-loss rates are high enough to affect GC ages seems remote.

### 2.3 *Uncertainty in $L^{\text{TO}}$ Due to Unconventional Physics*

In the late 1970s and early 1980s the possibility that the Gravitational constant  $G$  varied with time received considerable attention, due largely to the fact that this feature was common to three prominent cosmologies—Brans-Dicke (1961), Hoyle-Narlikar (1972a,b), and Dirac (1974). Some of the early tests of the implications of these theories for stellar evolution seemed to lead to satisfactory results (e.g. see Canuto & Lodenquai 1977, Vandenberg 1977, Maeder 1977). Even when potential difficulties, such as the apparent incompatibility of Dirac's theory with the observed characteristics of the microwave background, were pointed out (Steigman 1978), it was often possible to accommodate those objections by revising the theory (cf Canuto & Hsieh 1978). Using their flavor of gravitational theory, Canuto & Hsieh (1981) showed that it was possible for the estimated ages of GCs to decrease from 15 Gyr, under canonical assumptions, to  $< 10$  Gyr, if  $G$  varied at a rate ( $\dot{G}/G \approx -6 \times 10^{-11} \text{ yr}^{-1}$ ) that was consistent with observed limits at that time (see Van Flinders 1981).

However, those limits are now very much tighter. Taylor & Weisberg (1989) have determined that  $\dot{G}/G = (1.2 \pm 1.3) \times 10^{-11} \text{ yr}^{-1}$  from pulse time-of-arrival observations of the binary pulsar PSR1913+16 over the previous 14 years. Their data are completely consistent with Einstein's Theory of General Relativity. In addition, Müller et al (1991) obtain  $\dot{G}/G = (0.01 \pm 1.04) \times 10^{-11} \text{ yr}^{-1}$  from 20 years worth of lunar laser ranging data. These results essentially eliminate the possibility of temporal variations in  $G$  being a factor in the determination of GC ages.

More promising, perhaps, is the following idea: If nonbaryonic Weakly Interacting Massive Particles (or WIMPs) constitute the dark matter in the Universe, then they might be accreted by stars and affect their evolution (Steigman et al 1978, Press & Spergel 1985). Being massive, they would tend to collect in the cores of stars, and by virtue of being weakly interacting, they would provide an efficient means of central energy transport. If such particles resided in the Sun, for instance, they could lower the central temperature enough to enable a solution to the solar neutrino problem (Faulkner & Gilliland 1985, Spergel & Press 1985). This would require WIMP masses between approximately 2 and 7 GeV and interaction cross sections with nuclei within an order of magnitude (or so) of  $10^{-35} \text{ cm}^2$  [see Dearborn, Griest & Raffelt (1991), who also discuss recent experimental limits on these properties]. Furthermore, according to Faulkner & Swenson (1988, 1993), the deduced turnoff ages of globular clusters would be  $\sim 20\%$  less than canonical estimates, if their member stars acquired sufficient numbers of WIMPs to isothermize the innermost 10% of their masses.

The main testable prediction, as far as GCs are concerned, is that stars containing WIMPs will leave the main sequence somewhat sooner than canonical stellar models would predict, due to the isothermal core effect, and spend more time on the subgiant branch, because they have extra hydrogen to burn in the shell-narrowing phase. That is, an observed luminosity function should show an excess of subgiants and giants relative to the number of turnoff stars, if WIMP models are more realistic than standard calculations. Surprisingly, this is actually seen in the luminosity–function data for a number of GCs (see Stetson 1991, VandenBerg & Stetson 1991, Faulkner & Swenson 1993, Bolte 1994). However—and this poses a problem—these “anomalies” appear to be present in the observations of only the extremely metal-deficient clusters; i.e. the same ones that show strong evidence for progressive mixing along the RGB. As shown in Section 2.5, new observations for M5 appear to conform remarkably well to standard evolutionary predictions, as do the available luminosity–function data for 47 Tuc (see Bergbusch & VandenBerg 1992). One is tempted to think that something to do with the deep-mixing phenomenon, rather than WIMPs, is the more likely cause of the unexpected luminosity function features.

However, models have not yet been constructed for GC stars that incorporate the very detailed theory for the accretion (and evaporation) of WIMPs that has been developed by Gould (1990) and Gould & Raffelt (1990a,b). These models may predict something quite different from calculations that attempt to mimic the effects of WIMPs by imposing (albeit in a self-consistent way) an isothermal core structure on an otherwise normal stellar model. Thus further work is certainly warranted—even though there are other indications that the WIMP hypothesis faces an uphill battle. For instance, using the Gould/Gould-Raffelt theory, Turck-Chièze et al (1993) find that solar models containing WIMPs do not appear to satisfy helioseismic constraints as well as Standard Solar Models. Also, VandenBerg & Stetson (1991) have suggested that WIMPs would likely suppress the formation of convective cores in the  $\approx 1.3M_{\odot}$  turnoff stars in the old open cluster M67. If this happened, then the observed gap feature at  $M_V \approx 3.5$  (see the recent CMD by Montgomery, Marschall & Janes 1993) would not be produced. (It is possible, of course, that the density of WIMPs is much greater in the halo of the Galaxy than in the disk and that the evolution of the Sun and M67 stars would be little affected.) In addition, WIMPs may (Renzini 1987) or may not (Spergel & Faulkner 1988) cause difficulties for our understanding of the horizontal-branch phase of low-mass stars (also see Dearborn et al 1990).

It would be premature to conclude that WIMPs, or the very similar “halons” that Finzi (1991, 1992) has proposed, or other dark-matter candidates like axions (Peccei & Quinn 1977; Dearborn, Schramm & Steigman 1986; Isern, Hernanz

& Garcia-Berro 1992) do not affect stellar ages (if they exist). But neither can one give very serious consideration to the possibility that they do, at least at this time. There appears to be a number of difficulties for the WIMP hypothesis to overcome, and the other suggestions have simply not been adequately developed and tested to pose a serious challenge to standard stellar evolutionary theory.

#### 2.4 *Uncertainty in $L^{\text{TO}}$ Due to the Assumed Chemistry of Stars*

It has long been known that the predicted age of a star of a given mass depends on its initial helium and heavy-element abundances (e.g. Demarque 1967, Iben & Rood 1970). Even the special importance of the CNO elements for stellar ages was appreciated early on (e.g. Simoda & Iben 1968). This has driven a huge, ongoing effort by many observers to define the detailed run of chemical abundances in field and halo stars as accurately as possible. Thanks to that effort, we now know (for instance) that  $[C/Fe]$  and  $[N/Fe] \sim 0$  over  $0.3 \lesssim [Fe/H] \lesssim -2$  and that the elements synthesized by  $\alpha$ -capture processes (e.g. O, Ne, Mg, Si, etc) are enhanced, relative to iron, in metal-poor stars by a factor of 2–3 (see the comprehensive review by Wheeler, Sneden & Truran 1989). (In the standard notation, this corresponds to  $[\alpha/Fe] \approx 0.3\text{--}0.5$ , where  $\alpha$  represents O or Ne or Mg, etc.) It is not yet definite that all of the so-called  $\alpha$ -elements scale together as there is considerable scatter in the field star observations (some of it real): Thus, the precise shapes of the mean relations between the various [element/Fe] ratios as a function of [Fe/H] still have some degree of uncertainty. Also, whether or not field and GC dwarfs of the same iron content are chemically indistinguishable remains a matter of some concern. But the chemistry of stars appears to be largely under control.

High-resolution spectroscopy (e.g. Cohen 1979, Sneden et al 1991) and the tightness of observed CMDs (e.g. Stetson 1993; Folgheraiter, Penny & Grifiths 1993) have established that the dispersion in Fe abundances is very small in nearly all GCs ( $\omega$  Cen and possibly M22 being exceptions). Moreover, the spectroscopic data now yield [Fe/H] values that are accurate to within  $\approx \pm 0.2$  dex, if not better. According to the upper panel of Figure 4—which shows plots of the turnoff luminosity versus age relations that Vandenberg et al (1996) have computed for various choices of [Fe/H],  $[\alpha/Fe]$ , and  $Y$ —this implies an uncertainty in the age at a given  $M_{\text{bol}}(\text{TO})$  of about  $\pm 1$  Gyr ( $\approx \pm 7\%$ ). Furthermore, since the  $\alpha$ -element contents of stars in the extremely metal-deficient clusters like M92 appear to be within  $\pm 0.15$  dex of  $[\alpha/Fe] = 0.4$  (e.g. Sneden et al 1991; McWilliam, Geisler & Rich 1992), the corresponding age uncertainty is expected to be about  $\pm 4\%$  (judging from Figure 4). This makes a total

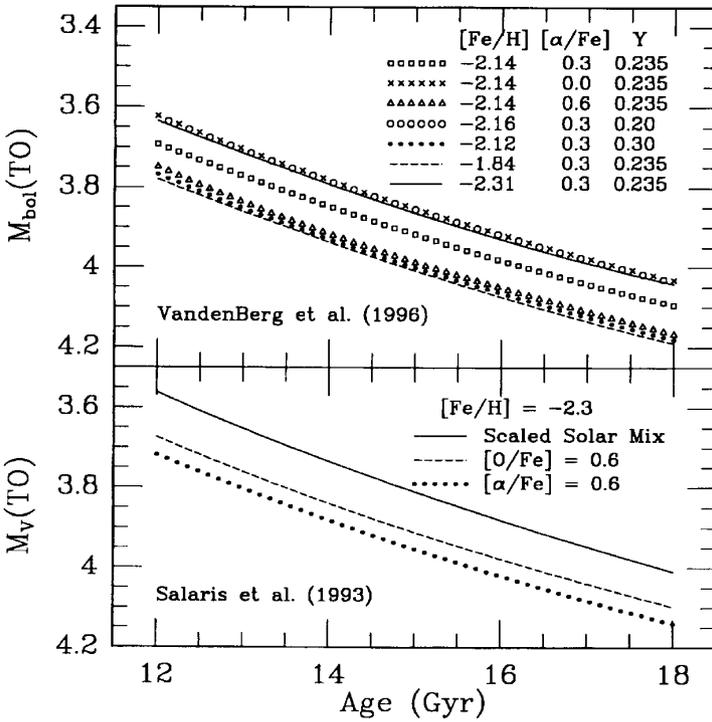


Figure 4 Turnoff luminosity vs age relations for various chemical composition parameters.

uncertainty of  $\pm 11\%$  in the turnoff ages due to current errors in heavy-element abundance determinations.<sup>4</sup>

Helium-abundance uncertainties could potentially affect age estimates at the few percent level (see the upper panel of Figure 4), but  $Y$  appears to be rather well determined, in spite of the fact that the methods used are indirect. [Spectral features due to helium can be detected in hot HB stars, but gravitational settling is known to be important in them (e.g. Heber et al 1986).] Foremost among

<sup>4</sup>At first sight, Figure 4 would appear to contradict the claim by Chieffi, Straniero & Salaris (1991) that enhancements in the  $\alpha$ -elements do not lead to younger ages for the GCs (also see Bencivenni et al 1991). But, in fact, the reason why they obtained similar ages using either  $\alpha$ -enhanced or scaled-solar abundance isochrones is that they set the distances to the globulars using theoretical horizontal-branch calculations, which predict that the HB luminosity should decrease as  $[\alpha/Fe]$  increases. Only by an appropriate adjustment of the GC distance scale is it possible to reach the conclusion that ages are insensitive to  $[\alpha/Fe]$ : The turnoff age-luminosity relations computed by Salaris, Chieffi & Straniero (1993) both for  $[\alpha/Fe] = 0.0$  and for  $[\alpha/Fe] > 0.0$  are very similar to those derived by Vandenberg et al (1996).

these techniques is the so-called R-method (Iben 1968b), which compares the ratio of the predicted HB and RGB lifetimes,  $t_{\text{HB}}/t_{\text{RGB}}$ , as a function of  $Y$ , with the observed number ratio of stars in these phases. Using mainly the calibration of Buzzoni et al (1983) (also see Caputo, Martinez Roger & Paez 1987), nearly all applications of the R-method (e.g. Buonanno, Corsi & Fusi Pecci 1985; Ferraro et al 1992, 1993) have yielded  $Y = 0.23 \pm 0.02$ . Discrepant results have been obtained for a few globulars, such as M68 (Walker 1994), for which the R-method implies  $Y \sim 0.17$ ; however, in that particular case, the analogous ratio of the numbers of asymptotic-giant branch to RGB stars gives an estimate of the helium abundance that is within  $1\sigma$  of  $Y = 0.23$ . (Why M68 has such an anomalous R value is presently unknown.)

Fits to the morphologies of observed HB populations (e.g. Dorman, Vandenberg & Laskarides 1989; Dorman, Lee & Vandenberg 1991) and to the red edges of the RR Lyrae instability strips in clusters (Bono et al 1995) reinforce the R-method results. Pulsation models have traditionally favored  $Y \approx 0.30$ , but due to the advent of the OPAL (Rogers & Iglesias 1992) and OP (Seaton et al 1994) opacities, lower values of  $Y$  can now be accommodated (Kovács et al 1992, Cox 1995). The adoption of  $Y \approx 0.23$  in models for GC stars is further supported by the fact that this value is very close to that predicted by standard and inhomogeneous Big Bang nucleosynthesis calculations (see, e.g. Krauss & Romanelli 1990 and Mathews, Schramm & Meyer 1993, respectively), as well as empirical determinations of the pregalactic helium abundance (Pagel et al 1992; Izotov, Thuan & Lipovetsky 1994; Olive & Steigman 1995).

We conclude this section by emphasizing the importance of oxygen to stellar age determinations. Plotted in the lower panel of Figure 4 are the age versus turnoff luminosity relations that Salaris et al (1993) have derived for  $[\text{Fe}/\text{H}] = -2.3$  and various assumptions about the element mix. This plot shows that most of the reduction in age at a given  $L^{\text{TO}}$  that results from an enhancement in the  $\alpha$ -elements is due to oxygen. Getting the oxygen abundance right is, therefore, a much bigger concern than having precise abundances for most of the other heavy elements. This result is not unexpected given the large abundance of oxygen and its role as a catalyst in the CNO-cycle and as a major contributor to bound-free opacities in stellar interiors (see, e.g. Vandenberg 1992).

## 2.5 Tests of Stellar Models

The interior structures of low-mass, main-sequence stars are believed to be much simpler, and therefore (presumably) better understood, than those of their higher-mass ( $M \gtrsim 1.15 M_{\odot}$ ) counterparts because, in part, they do not contain convective cores and so are unaffected by the uncertainties (e.g. the extent of overshooting) associated with them. Perhaps the main evidence for

possible inadequacies in the theory has been the longstanding failure of canonical models to reproduce the observed flux of neutrinos from the Sun, but solar oscillation studies have considerably diminished that concern. As Dziembowski et al (1994, 1995) have concluded, the inferred structure of the Sun from helioseismology is now so close to that predicted by the standard model, throughout its interior, that there is little room left for an astrophysical solution to the solar neutrino problem. Certainly, there are many examples in the scientific literature demonstrating how well current stellar evolutionary theory can match superb observational data. One of the nicest of these is the study of the Hyades by Swenson et al (1994). They obtained a self-consistent fit to the CMD, to the mass-luminosity relation defined by the cluster binaries, and to the Li abundances in the G stars, using opacities for the observed [Fe/H] value and without applying any ad hoc adjustments of any kind. Indeed, the *best-observed* binaries (e.g. AI Phe—see Andersen et al 1988) and the mass-luminosity relations derived from them appear to agree rather well with the predictions of standard models (cf Andersen 1991).

Considering the more evolved, post-turnoff phases, the main challenge to the theory would appear to be the observed chemical abundance variations among bright GC giants (already summarized in Section 2.2.2) and some anomalies in the luminosity function (LF) data for a few clusters (see below). These difficulties will probably be resolved once rotation is accurately treated (which, admittedly, is not easily done). Otherwise, as extensively reviewed by Renzini & Fusi Pecci (1988), there appears to be little basis for believing that the sorts of models that have been computed for the past 25 years or so are seriously in error. Although many discrepancies between theory and observation can be identified, it is much more likely that they are due to deficiencies in, for instance, the opacity or convection theory, than to a problem with the basic stellar structure equations themselves. But it may be the case that the LF anomalies have a different origin.

Relatively little work has been done on the luminosity functions of GCs, in spite of the fact that they provide a superior test of stellar models compared with the fitting of CMDs and despite some tantalizing results from early studies. For instance, Simoda & Kimura (1968) suggested that the LFs of M3 and M13 differed from one another—which might be an important clue (yet to be followed up) as to the cause of the differences in the HB morphologies of these common-[Fe/H] clusters. Also, making use of the fact that LFs provide one of the few ways to infer the helium abundances in GCs (see the recent study by Ratcliff 1987), Hartwick (1970) derived  $Y \sim 0.35$  from such an analysis of M92. However, not until Bergbusch's (1990) study of the latter cluster was a possible inconsistency between an observed LF and theoretical predictions

identified. Depending on how the synthetic and observed LFs were matched, the M92 data showed either a broad dip between  $19 < V < 20$  or a bump near  $V = 18.6$  that was not present in the models. Stetson's (1991) combined LF for M68, M92, and NGC 6397, based on new CCD observations of these clusters, confirmed the existence of these features. In addition, it revealed that, when theoretical LFs for the appropriate  $Y$  and  $Z$  were normalized to the turnoff data, the observed RGB had a significant excess of stars compared with the number predicted. These anomalies are not readily explained in terms of variations in any of the usual parameters, but they have turned out to be the anticipated signature of WIMPs (as already recounted in Section 2.3).

The lower panel in Figure 5 illustrates Bolte's (1994) LF for M30, and it too poses the same problems for the theory as the data for the other [Fe/H]

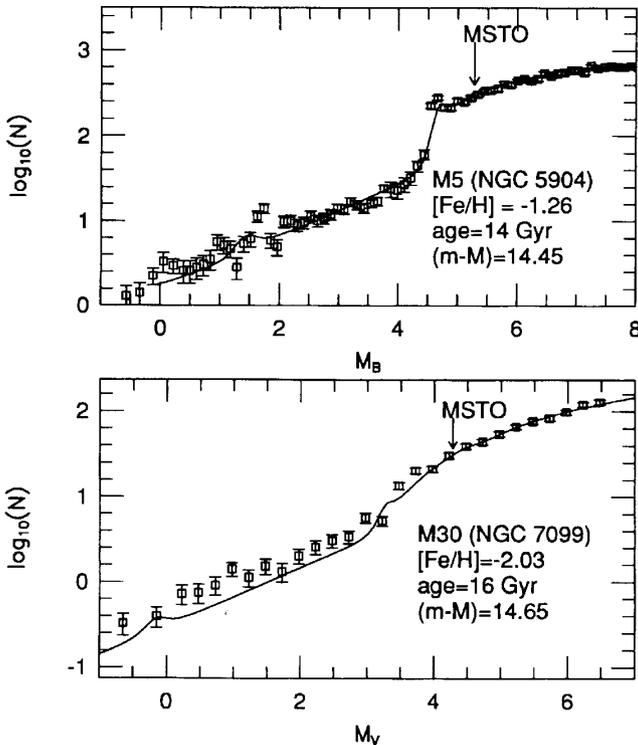


Figure 5 Comparisons of the observed luminosity functions for M5 (upper panel) and M30 (lower panel) with Bergbusch & Vandenberg (1992) isochrones, on the assumption of the indicated ages, [Fe/H] values, and distance moduli. The M5 data are from Sandquist et al (1996); the M30 results are as reported by Bolte (1994).

$\sim -2.1$  GCs. But note that the M5 LF (in the *upper panel*) shows no such anomalies; indeed, it conforms remarkably well to the theoretical predictions. Similarly, Bergbusch & Vandenberg (1992) have not found any obvious difficulties in fitting the available luminosity function data for 47 Tuc (though their matching of the brighter to the fainter data is somewhat uncertain). And Stetson & Vandenberg's (1996) Canada-France-Hawaii Telescope photometry for a sample of  $\sim 10^5$  stars in M13 shows no evidence of a subgiant bump either. Curiously, their very preliminary analysis suggests that the RGB in M13 may be underpopulated relative to the turnoff; i.e. the opposite to what is seen in the more metal-poor clusters.

There is clearly much to be learned from such LF studies, but from these first results, one has the impression that the anomalous subgiant bump is characteristic of only the extremely metal-poor clusters—and hence it can hardly be due to WIMPs, which should not show a preference for a particular [Fe/H] value. Is that feature somehow connected with the deep-mixing phenomenon? We do not know. The differences in the relative RGB-to-turnoff populations might be due to differences in helium abundance. Alternatively, it may be an indication of differences in core rotation. Using the simplest possible treatment of rotation (cf Mengel & Gross 1976), Larson, Vandenberg & De Propris (1995) have found that the number of giants relative to the number of turnoff stars is larger if the stars have significant internal rotation. Perhaps the main point to be made here is that, unless and until the LF data are satisfactorily explained, one should be wary of trusting the application of standard, nonrotating, unmixed-envelope models to those clusters (apparently the most metal-poor ones) whose luminosity functions cannot be reproduced by such models.

We conclude that, although there remain unexplained observations of evolved stars in globular clusters, the stellar models for GC stars at the main-sequence turnoff and probably to a few magnitudes down the presently observed main sequence are reliable. In particular, the agreement between predicted and observed mass-luminosity relations suggests that the theory is basically correct and essentially complete. The main outstanding issue for models (in the context of predicting the main-sequence lifetimes of low-mass stars) is the extent to which helium diffusion may reduce cluster age estimates. Our best estimate for the maximum reduction in ages due to this effect is  $\lesssim 1$  Gyr. Although it could be postulated that some currently unconsidered physics will in the future reduce measured cluster ages, there currently does not appear to be any viable candidate physical processes, nor is there any clear motivation for seeking them other than to try to reduce GC ages.

## 2.6 *The First Estimates of Globular Cluster Ages*

It is instructive to look back to the first papers that were written on the subject of globular cluster ages. Using hand computation, Sandage & Schwarzschild (1952) produced the first evolutionary tracks for low-mass Population II stars to somewhat beyond central hydrogen exhaustion. From these calculations they inferred an age of  $3.5 \times 10^9$  yr for the two globulars whose turnoffs had just been detected—M92 (Arp, Baum & Sandage 1953) and M3 (Sandage 1953). However, they had not modeled the earliest, low-luminosity phases, and when this was taken into account, the estimated age rose to  $6.2 \times 10^9$  yr (Hoyle & Schwarzschild 1955). Essentially the same result (6.5 Gyr) was obtained by Haselgrove & Hoyle (1956), who were the first to use a digital computer to solve the stellar structure equations.

In the 1950s, it was generally supposed that the original matter in the Galaxy was pristine (i.e. “uncooked”); consequently, the stellar models that were computed at that time assumed  $Y \approx 0.0$ . It was not realized until somewhat later that it would be very difficult for conventional stellar nucleosynthesis to explain the increase from such low  $Y$  values to the observed high helium contents of Population I stars (see Hoyle & Tayler 1964), and it was later still that the microwave background was discovered (Penzias & Wilson 1965) and the notion that the Universe began as a singularity took hold. Big Bang nucleosynthesis calculations carried out shortly thereafter (e.g. Wagoner, Fowler & Hoyle 1967) predicted that the primordial helium abundance would be somewhere in the range  $0.2 \leq Y \leq 0.3$ .

However, even before these developments, Hoyle (1959) had computed an evolutionary track for  $Y = 0.249$  and  $Z = 0.001$  to explore the consequences of higher  $Y$ . Because that  $Y, Z$  combination is very close to what is assumed in present-day stellar models, we thought that it would be interesting to compare the track that Hoyle computed (as tabulated in his paper) with one for the same mass ( $1.163 M_{\odot}$ ) and chemical composition using the latest version of the University of Victoria code (see Vandenberg et al 1996). That comparison is shown in Figure 6. Considering the primitive state of our understanding of stellar physics nearly 40 years ago—even the relative importance of the  $pp$ -chain versus the CNO-cycle was largely unknown—the agreement is remarkably good. The turnoff temperatures agree to within 240 K, the turnoff luminosities to within  $\Delta(\log L/L_{\odot}) = 0.05$ , and the turnoff ages to within  $\approx 40\%$  (4.3 Gyr for Hoyle’s model versus 2.7 Gyr for ours).

The point of this exercise is to show that the first stellar models computed for GC stars predicted a higher, not a lower, age at a fixed turnoff luminosity than do modern calculations. (The adoption of  $Y \approx 0.0$  would tend to further increase that age.) Therefore, the low ages reported in those initial investigations

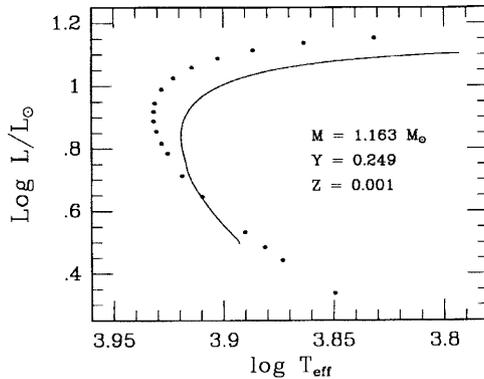


Figure 6 Comparison of an evolutionary track computed by Hoyle (1959) (*dotted curve*) with one for the same mass and chemical composition, as specified, but using the latest version of the University of Victoria code (*solid curve*).

must be attributed to something other than the evolutionary models that were used. In fact, they resulted from the then conventional assumption that the RR Lyrae variables, which were used to set the GC distance scale, had  $M_V = 0.0$ . Only after such studies as that by Eggen & Sandage (1959), who used trigonometric parallax stars encompassing a range in  $[\text{Fe}/\text{H}]$  and the nearby Groombridge 1830 group of low-metallicity subdwarfs to do main-sequence fits, did it become accepted that the actual luminosities of the RR Lyraes must be near  $M_V = 0.5$  [although there were earlier indications that this might be the case (e.g. Pavlovskaja 1953)]. Hoyle (1959) noted that this revision would imply an age for the Galaxy of greater than  $10^{10}$  years. Thus, ages much more similar to current estimates would have resulted had the cluster distances been known more accurately. Even today, as we show in the next section, distance uncertainties continue to dominate over all other sources of error.

### 3. GLOBULAR CLUSTER DISTANCES

Given that the most reliable indicator of a globular cluster's age is its turnoff luminosity, the determination of precise distances to these systems is arguably the single most crucial observational input into the evaluation of accurate ages (see, e.g. Renzini 1991, Chaboyer 1995, Bolte & Hogan 1995). Nearly everything that we know about GC distances is based on two standard candles—namely, the nearby subdwarfs and the RR Lyrae variable stars. Thanks to the development of the *Hubble Space Telescope* (*HST*), we will soon be able to add

white dwarfs to this very short list. These stars have the advantage of being essentially free of metallicity and convection complications (cf Fusi Pecci & Renzini 1979), and local white dwarf calibrators are much more numerous than subdwarfs. Although we can anticipate that the fitting to white dwarf cooling sequences will involve a number of difficulties (some unanticipated), it is encouraging that the first *HST* results, for M4 by Richer et al (1995), indicate a distance very similar to the one adopted by Richer & Fahlman (1984) on the assumption that  $M_V(\text{HB}) = 0.84$ . These results lead to their determination of an age of 13–15 Gyr for this cluster. We also recognize the potential of direct astrometric methods (see Cudworth & Peterson 1988, Rees 1992) and the existence of a number of other approaches (e.g. using the RGB tip magnitude) to constrain cluster distances. We, however, restrict the present discussion to the two classical distance calibrators.

### 3.1 *Subdwarf-Based Distances*

The nearby subdwarfs—metal-poor stars with halo kinematics whose orbits have brought them near enough to the Sun for them to have measurable trigonometric parallaxes—play two critical roles in the measurement of GC ages. First, with well-determined values of  $M_V$ , these objects provide a direct test of the model predictions for the position of the zero-age main-sequence as a function of [Fe/H] in the low-metallicity regime. Second, under the (testable) assumption that the subdwarfs are local versions of the unevolved main-sequence stars in globular clusters, they can be used to tie the cluster distances directly into the most reliable distance scale that exists in extra-Solar-system astronomy (that defined by trigonometric parallaxes).

The recognition of the importance of the subdwarfs and of their relation to the RR Lyraes and the halo GCs is itself an interesting story (see the review by Sandage 1986). An important landmark was Sandage's (1970) identification of eight subdwarfs with sufficiently good  $\pi$  measures for them to be useful for deriving the distances to GCs. He also used them to calibrate the absolute magnitude of the horizontal branch at the position of the instability strip in M3, M15, and M92. Carney (1979) and Laird, Carney & Latham (1988) improved the [Fe/H] determinations of that sample. In the pre-CCD era of photometry, however, the subdwarfs were of limited usefulness for establishing the Population II distance scale because of fairly large random and (in retrospect) scale errors in the measurement of faint main-sequence cluster stars [see, e.g. Figure 4 in Fahlman, Richer & Vandenberg (1985) and Figure 30 in Stetson & Harris (1988)]. CMDs derived from CCD data, beginning in the mid-1980s, made the adoption of a subdwarf-based distance scale a much more viable alternative to purely HB-based distance estimates. With CCDs and 4-m telescopes, the main sequences of nearby clusters could be defined very accurately down to

$M_V \sim 10$  (e.g. see Figure 36 in Stetson & Harris 1988). In the CCD era, the limiting factors in the derivation of cluster distances via subdwarf fitting became the scatter in the Population II main-sequence fiducial defined by the subdwarfs and the lingering uncertainties in the reddening and color calibrations of the cluster data.

Table 1 contains our compilation of relevant data for all stars in the 1991 edition of the Yale Trigonometric Parallax Catalogue with  $\sigma_\pi/\pi < 0.5$  and spectroscopic measures of  $[\text{Fe}/\text{H}] \lesssim -1.3$ . This list includes the original eight stars from Sandage (1970) minus HD 140283, which appears to be an evolved star (Magain 1989, Dahn 1994), plus an additional eight stars, which generally have large  $\sigma_\pi$  values. The tabulated  $\sigma_\pi$  values were taken from the Yale Catalogue; the apparent colors and magnitudes are from the compilation given in the Hipparcos Input Catalogue (Turon et al 1992). The absolute magnitudes were calculated from the usual equation:  $M_V = V + 5 + 5 \log(\pi)$ . Because trigonometric parallax measurements are subject to a Malmquist-like bias, arising from the coupling of the measuring errors with the steep slope of the true parallax distribution, there is a tendency for the observed parallaxes to be larger than their actual values. (This is true in the statistical sense for entire catalogues as well as for individual measurements.) The resultant so-called Lutz–Kelker (or L-K) corrections (Lutz & Kelker 1973) were determined to compensate for this effect. To be specific, we have applied the correction  $\delta M_V = -5.43(\sigma/\pi)^2 - 25.51(\sigma/\pi)^4$ , according to the formulation of Hanson (1979), who used the distribution of proper motions of objects in the parallax

**Table 1** Subdwarfs with  $\pi$  and  $[\text{Fe}/\text{H}]$  determinations

ID	[Fe/H]	V	B - V	$\pi$ (")	$\sigma_\pi$ (")	$M_V$	$\sigma(M_V)$	$M_V(\text{L-K})$	$(B - V)_{-2.14}$
HD 7808	-1.78	9.746	1.008	0.0663	0.0126	8.854	0.412	8.624	0.974
HD 19445	-2.08	8.053	0.475	0.0252	0.0052	5.060	0.448	4.783	0.471
HD 25329	-1.34	8.506	0.863	0.0548	0.0047	7.200	0.186	7.159	0.800
HD 64090	-1.73	8.309	0.621	0.0405	0.0023	6.346	0.123	6.328	0.591
HD 74000	-2.20	9.62	0.43	0.0155	0.0048	5.572	0.672	4.816	0.434
HD 84937	-2.12	8.324	0.421	0.0280	0.0064	5.560	0.496	5.206	0.420
HD 103095	-1.36	6.442	0.754	0.1127	0.0016	6.702	0.031	6.701	0.693
HD 134439	-1.4	9.066	0.770	0.0365	0.0025	6.877	0.149	6.851	0.714
HD 134440	-1.52	9.445	0.850	0.0365	0.0025	7.256	0.149	7.230	0.804
HD 149414	-1.39	9.597	0.736	0.0281	0.0035	6.841	0.270	6.750	0.679
HD 194598	-1.34	8.345	0.487	0.0194	0.0014	4.784	0.157	4.755	0.424
HD 201891	-1.42	7.370	0.508	0.0325	0.0027	4.929	0.180	4.891	0.462
HD 219617	-1.4	8.160	0.481	0.0280	0.0055	5.396	0.426	5.148	0.431
BD+66 268	-2.06	9.912	0.667	0.0216	0.0026	6.584	0.261	6.500	0.661
BD+11 4571	-3.6	11.170	1.060	0.0316	0.0047	8.668	0.323	8.536	1.080

catalogues to estimate the magnitudes of the L-K corrections. This expression for  $\delta M_V$  is strictly valid only for  $\sigma_\pi/\pi < 0.33$ . (The always-negative L-K corrections are added to the  $M_V$  values because the true luminosities are larger than the uncorrected estimates.)

The last column in Table 1 contains the predicted color that each star would have if its metallicity were  $[\text{Fe}/\text{H}] = -2.14$  (chosen to illustrate the subdwarf-fitting procedure for the specific case of M92). At a fixed mass, main-sequence stars of different  $[\text{Fe}/\text{H}]$  will encompass a range in color and  $M_V$ ; consequently, to define a fiducial for distance determinations by the main-sequence fitting technique using subdwarfs, it has become common practice to derive a mono-metallicity subdwarf sequence. This is obtained by correcting the color of each subdwarf, at its observed  $M_V$ , by the difference between the predicted colors of stars with the  $[\text{Fe}/\text{H}]$  of the subdwarf and that of the cluster itself. Thus, the model colors are used only differentially. Bi-cubic interpolation through a table of  $B - V$  colors at different  $[\text{Fe}/\text{H}]$  and  $M_V$  values, generated from the Bergbusch & Vandenberg (1992) isochrones, was used to generate the color corrections: These take into account the dependence of radius on metallicity at fixed luminosity as well as purely atmospheric line blanketing effects.

Figure 7 shows how well 16 Gyr,  $[\alpha/\text{Fe}] = 0.3$  isochrones for  $[\text{Fe}/\text{H}] = -1.31, -1.71, \text{ and } -2.14$  (from Vandenberg et al 1996<sup>5</sup>) coincide with the positions of the local subdwarfs on the color-magnitude plane. We have plotted all of the stars in Table 1 (specifically, the data in the fourth, eighth, and ninth columns) whose metallicities fall within  $\pm 0.15$  dex of the isochrone  $[\text{Fe}/\text{H}]$  values. The agreement is about as good as one could hope for. Note, in particular, how well the models satisfy the constraint provided by the best of the subdwarfs (HD 103095, also called Groombridge 1830) and that the lower metal abundance subdwarfs tend to be displaced from those of higher  $Z$  in roughly the direction and amount suggested by the theory.

A main-sequence fit of M92 to the subdwarfs, using the data in the eighth, ninth, and tenth columns of Table 1 for those stars with  $\sigma(M_V) < 0.3$  mag, is illustrated in Figure 8. When a foreground reddening of 0.02 mag (see Stetson & Harris 1988) is assumed, an apparent distance modulus of 14.65

<sup>5</sup>We make fairly extensive use of these calculations in this study, obviously because they are immediately at hand, but also because they represent the most up-to-date models presently available. In particular, they employ opacities for the adopted  $\alpha/\text{Fe}$  number abundance ratios and are not based on the renormalization of scaled-solar-mix calculations, as has been advocated by Salaris et al (1993). Their procedure does appear to work well at low  $Z$  values, but not for  $Z > 0.002$  (or so) according to Vandenberg et al (1996; also see Weiss, Peletier & Matteucci 1995): At high  $Z$ , the RGB location becomes insensitive to  $[\alpha/\text{Fe}]$ . Importantly, as shown by Vandenberg (1992), Salaris et al, and the three lowermost curves in Figure 1 of this paper, virtually identical results are obtained when completely independent codes employing similar physics are used.

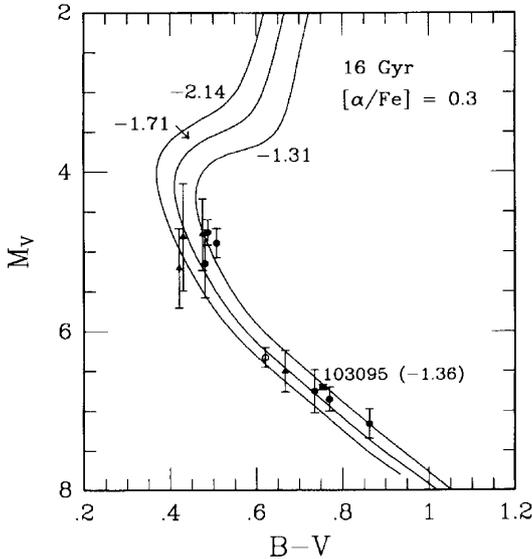


Figure 7 Comparison of the CMD locations of the nearby subdwarfs, whose properties are in the fourth, eighth, and ninth columns of Table 1, with Vandenberg et al (1996) isochrones. The closed circles, open circle, and closed triangles represent those subdwarfs whose tabulated  $[\text{Fe}/\text{H}]$  values are within  $\pm 0.15$  dex of those of the three isochrones; namely,  $-1.31$ ,  $-1.71$ , and  $-2.14$ , respectively. All of the isochrones assume  $[\alpha/\text{Fe}] = 0.3$  and an age of 16 Gyr, though the latter choice is inconsequential.

mag is obtained. The Vandenberg et al (1996) isochrones, for the indicated parameters, have simply been overlaid on this figure, i.e. no color adjustments of any kind have been applied to them. [Their temperature and color scales are very close to those of the Bergbusch & Vandenberg (1992) calculations, which were used to produce the  $B - V$  data in the last column of our table.] One has the impression that a small redward color shift should be applied to the isochrones at the fainter magnitudes, but what differences exist are clearly small.

An age of 15.5–16 Gyr is indicated from the observed location of the turnoff and subgiant branch relative to their theoretical counterparts. Allowing for helium diffusion would reduce this estimate to  $\approx 15$  Gyr (see Section 2.2.1), which should not be in error by more than  $\pm 1.5$  Gyr due to chemical composition uncertainties (see Section 2.4). According to Section 2.1.3, it is possible that deficiencies in convection theory could contribute a small age uncertainty, but other than this minor concern, remaining uncertainties in stellar physics should have little impact. Assuming no systematic error in the distance scale defined by the L-K corrected trigonometric parallax measures, the M92

distance modulus error is dominated by three terms [see Stetson & Harris (1988) for a more complete discussion of the errors associated with the subdwarf fit]. There is a goodness-of-fit term, which we approximate with the RMS vertical scatter (after correcting the colors to  $[Fe/H] = -2.14$ ) of the subdwarf distribution around the distance-modulus-adjusted M92 main-sequence; a term for the uncertainty in the reddening towards M92,

$$\delta E(B - V) \times \frac{\partial M_V}{\partial (B - V)};$$

and a term for the uncertainty in the  $[Fe/H]$  value for M92 stars,

$$\delta [Fe/H] \times \frac{\partial M_V}{\partial (B - V)} \times \frac{\partial (B - V)}{\partial [Fe/H]}.$$

If we take  $\delta E(B - V) \sim 0^m.02$  and  $\delta [Fe/H] \sim 0.2$  dex, then these three terms added in quadrature give  $\sigma(m - M) \sim 0^m.16$ , which translates into an uncertainty in the age of  $\sim 2.0$  Gyr (68% confidence interval). The  $\delta [Fe/H]$

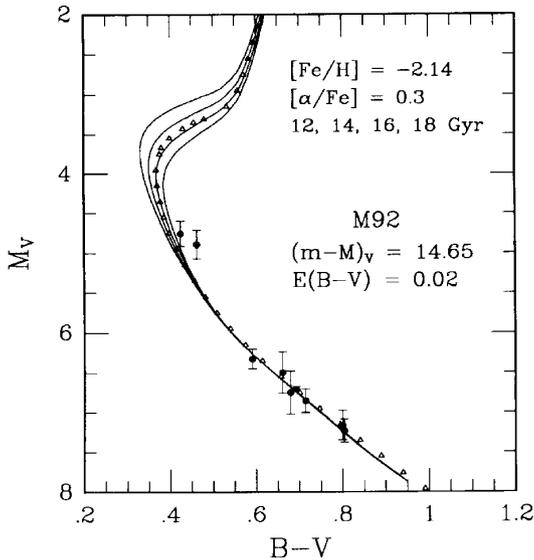


Figure 8 Main-sequence fit of the Stetson & Harris (1988) M92 main-sequence fiducial (*open triangles*) to the subdwarfs (*closed circles*), after the colors of the latter have been adjusted to compensate for differences between their  $[Fe/H]$  values and that of the cluster (see text). These revised colors are as given in the last column of Table 1. Only those data for which  $\sigma(M_V) < 0.3$  mag have been plotted. Vandenberg et al (1996) isochrones for the indicated chemical composition and ages have been overlayed onto (not fitted to) the observations.

term enters the age uncertainty a second time because  $M_{\text{bol}}(\text{TO})$  has an  $[\text{Fe}/\text{H}]$  dependence, and the formal *observational* uncertainty in the age that we derive for M92, *assigning no errors to the models and assuming the subdwarf distances have no systematic errors*, is 2.2 Gyr.

### 3.2 Distances Based on RR Lyraes

As noted in Section 2.6, the level of the horizontal branch at the color of the instability strip, which we refer to as  $M_V(\text{HB})$  (although any bandpass can be used), has been used for  $\sim 40$  years to set the distance to a globular cluster. This value is defined to be the mean absolute magnitude of the cluster RR Lyrae stars after a proper averaging over each star's pulsational cycle. However, because horizontal-branch stars evolve to brighter magnitudes on their way to the asymptotic giant branch, there is an evolutionary width in the brightness of the HB (see Sandage 1990a); consequently, when comparing literature data for  $M_V(\text{HB})$ , care must be taken to ensure that the level of the HB is being compared for stars that have undergone the same amount of evolution.

It has long been suspected (cf Sandage 1958) that  $M_V(\text{HB})$  is a function of  $[\text{Fe}/\text{H}]$ —considering equivalent evolutionary states—in the sense that more metal-poor HB stars are more luminous. As a result, a linear relation of the form  $M_V(\text{HB}) = c_0 + c_1 [\text{Fe}/\text{H}]$  has generally been assumed, and a concerted effort has been made to try to determine the constants  $c_0$  and  $c_1$ . The first of these constants is of critical importance for determining the age of the oldest GCs, while the second has a strong influence on the inferred age-metallicity relation that describes these systems (see, e.g. Sandage & Cacciari 1990, Walker 1992). Unfortunately, even the nearest RR Lyrae is too far away for a direct trigonometric parallax distance; consequently, it has been necessary to use more indirect approaches to determine their  $M_V$  values. These include statistical parallaxes of field variables (e.g. Hawley et al 1986, 1996), Baade–Wesselink (B-W) analyses of field and cluster RR Lyraes (e.g. Liu & Janes 1990a,b; Storm, Carney & Latham 1994), main-sequence fits to GCs (Buonanno et al 1990, Bolte & Hogan 1995), and pulsation theory (Sandage, Katem & Sandage 1981; Sandage 1993b).

Figure 9 provides a graphical summary of the current status of this endeavor. The filled squares give the B-W results of Jones et al (1992), supplemented by an additional two stars from earlier work by Liu & Janes (1990a) that were not considered by the former. (Both investigations analyzed essentially the same sample of field RR Lyraes and both obtained very similar  $M_V$  values, generally agreeing to within 0.03 mag.) The solid curve gives the linear fit to these data adopted by Storm et al (1994); specifically,  $M_V(\text{HB}) = 1.02 + 0.16 [\text{Fe}/\text{H}]$ . The slope of this relation is very similar to that of the dotted line, which represents the variation that Dorman (1993) computed from his zero-age main-sequence

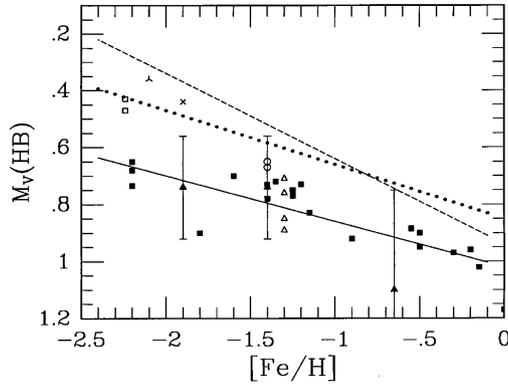


Figure 9  $M_V(\text{HB})$  vs  $[\text{Fe}/\text{H}]$  results from various sources. The closed squares represent the data for individual field RR Lyraes as obtained from the Baade-Wesselink analyses of Jones et al (1992) and Liu & Janes (1990a). The solid line gives the fit to these observations that was adopted by Storm et al (1994). Also based on the B-W method, the Liu & Janes (1990b) findings for four M4 variables are indicated by open triangles and the Storm et al results for RR Lyraes in M5 and M92 are denoted by open circles and open squares, respectively. The statistical parallax determinations by Hawley et al (1995) are represented by the closed triangles with attached error bars. The cross indicates Walker's (1992a) LMC estimate. The three-pointed star depicts the mean magnitude of M15 RR Lyraes as determined by Silbermann & Smith (1995). The dotted line gives the predicted ZAHB relation, as computed by Dorman (1993). The dashed line illustrates the relation between RR Lyrae magnitudes and  $[\text{Fe}/\text{H}]$  derived by Sandage (1993b) from his analysis of the Oosterhoff-Arp period-metallicity relation.

(ZAHB) models for scaled-solar abundances. It has  $c_0 = 0.85$  and  $c_1 = 0.19$ , which are exceedingly close to the coefficients that Renzini (1991) determined from the ZAHBs in Sweigart, Renzini & Tornambè (1987).

Why the B-W  $M_V$  values are fainter, at a given  $[\text{Fe}/\text{H}]$ , than the theoretically predicted values is hard to explain unless 1. the field RR Lyraes have a helium abundance that is significantly lower than the  $Y \approx 0.23$  assumed in the models, 2. the application of the B-W method introduces a 0.2 mag zero-point error, or 3. the stellar interior computations are somehow deficient. The first option seems improbable in view of the  $R$ -method results and the present consensus that the primordial helium abundance was near  $Y = 0.23$  (see Section 2.4). Concerning the last option, the only possibility that occurs to us is that the reduction in the envelope helium abundance due to diffusion has been underestimated (see Section 2.2.1). (Smaller core masses might also work, but there is no other reason to doubt our present understanding of the neutrino emission processes that largely determine the thermal structure of the core during RGB evolution.) Otherwise, any noncanonical process that might be going on, such as deep

mixing or rapid core rotation, would tend to make the models brighter rather than fainter. A zero-point error could well be the most probable solution given that Carney, Storm & Jones (1992) themselves suspect that  $c_0$  has a  $\pm 0.15$  mag uncertainty (also see Cohen 1992, Fernley 1994). This question awaits a satisfactory resolution.

Statistical parallaxes of field RR Lyrae stars also give a faint value for the HB brightness zero-point, but with a very large uncertainty. (This approach is subject to the assumption that the kinematic properties of the RR Lyrae population are, to first order, constant over the volume sampled.) Although the studies of Hawley et al (1986), Barnes & Hawley (1986), and Strugnell, Reid & Murray (1986) treated the existing data with a sophisticated set of analysis tools, there have remained some question marks regarding systematic errors in the proper motion lists available at the time and the use of a heterogeneous mix of [Fe/H] and radial velocity data. However, the recent reanalysis of the statistical parallax solution using a homogenous set of proper motion data based on an extragalactic coordinate system and new observations of [Fe/H] and radial velocity (Hawley et al 1995) has yielded essentially the same brightness zero-point as the earlier studies. (Note the location of the closed triangles with attached error bars in Figure 9.)

Beginning with the Cohen & Gordon (1987) investigation, there have been a number of attempts to carry out B-W analyses of cluster (as opposed to field) RR Lyraes. The open symbols in Figure 9 represent the results that Liu & Janes (1990b) obtained for M4 along with those derived for M5 and M92 by Storm et al (1994). (There has been a considerable evolution in the application of the B-W method over the years with the switch from  $BV$  to near-infrared photometry, the recognition that certain phases of the light curves give inconsistent results due to well-understood violations of assumptions, and the development of improved procedures for fitting the data. Thus only the latest determinations have been included in our figure.) Whereas the M4 RR Lyraes appear to be completely consistent with the field-star relation between  $M_V(\text{HB})$  and [Fe/H], that is apparently not true of the two M92 pulsators, and possibly not of the M5 variables, although Storm et al (1994) suggest that the difference is not significant in the latter case.

Based on the two RR Lyraes studied, Storm et al (1994) derived  $(m - M)_0 = 14.60 \pm 0.26$  for M92, i.e. effectively the same distance that Stetson & Harris (1988), Bolte & Hogan (1995), and we (in Section 3.1) obtained from the fitting of the nearby subdwarfs to the cluster main sequence. Storm et al expressed the concern that the two M92 variables might be highly evolved from the ZAHB, which would explain their displacement from the field-star relation, but there is additional evidence in support of the brighter luminosity scale. From CCD

observations of 182 RR Lyraes in seven Large Magellanic Cloud clusters, Walker (1992) determined a mean  $M_V$  of 0.44 mag at  $[\text{Fe}/\text{H}] = -1.9$  (the cross in Figure 9), assuming the Cepheid-based distance modulus of 18.5 (which should be accurate to within  $\pm 0.1$  mag). Curiously, Walker (1989) found the field RR Lyraes in the vicinity of the LMC cluster NGC 2257 to be 0.17 mag fainter, in the mean, than the cluster variables. Although this could simply be telling us that the cluster is closer than the average distance of the field stars, it could also be indicating a fundamental difference between the two stellar populations (a possible interpretation of the M92 results, as well). In addition, we note the determination of  $M_V = 0.36 \pm 0.12$  (the three-pointed star in Figure 9) for the M15 variables from an analysis of their pulsational properties (Silbermann & Smith 1995).

Lastly, there is the Sandage (1993b) relation,  $M_V(\text{HB}) = 0.94 + 0.30 [\text{Fe}/\text{H}]$  (the dashed curve in Figure 9), which is based on his analysis of the Oosterhoff-Arp period versus metallicity correlation: Oosterhoff (1939, 1944) showed that GCs separate into two groups according to the mean periods of their respective RR Lyrae populations, while Arp (1955) discovered that the separation was one of cluster metal abundance. Based on several pieces of evidence, Sandage (1993a) concluded that the Oosterhoff-Arp effect is well described by  $d \log P / d[\text{Fe}/\text{H}] = -0.12 \pm 0.02$ , where  $P$  is the mean period (in days) of the *ab*-type RR Lyraes. Then, using the fundamental pulsation equation,  $P\sqrt{\bar{\rho}} = \text{constant}$ , which can be turned into an equation in which  $P$  is given as a function of the pulsator's mass, luminosity, and effective temperature—namely,

$$\log P = 0.84 \log L/L_{\odot} - 0.68 \log \mathcal{M}/\mathcal{M}_{\odot} - 3.48 \log T_{\text{eff}} + 11.502$$

(van Albada & Baker 1973)—he inferred that the relation between  $M_V(\text{HB})$  and  $[\text{Fe}/\text{H}]$  must be steeply sloped, with the most metal-poor variables having rather bright magnitudes. This result made use of his deduction (in Sandage 1993b) that the instability strip is shifted towards cooler temperatures by  $\Delta \log T_{\text{eff}} = 0.012$  for each dex decrease in  $[\text{Fe}/\text{H}]$ . Thus, he contended that both a luminosity *and* a temperature shift must be taken into account to explain the observed period data.

It is unfortunately the case that the pulsation period depends sensitively on  $T_{\text{eff}}$  (see above), which is always very difficult to determine reliably. Prior to the Sandage (1993a,b) papers, a concerted effort had been made (see, e.g. Sandage 1982; Gratton, Ortolani & Tornambè 1986; Lee et al 1990; Sandage 1990b; Carney et al 1992; Catelan 1992, 1994; and references therein) to understand the so-called period-shift phenomenon, which is the term given to the dependence of pulsation period on metallicity at fixed amplitude, subsequently taken to be fixed  $T_{\text{eff}}$ . If canonical stellar models for  $Y = 0.23$  (or so) are read at fixed  $T_{\text{eff}}$ , they are unable to produce a significant period shift between, for instance, M3

and M15 (whose observations have been central to this issue), *if* the RR Lyraes are near their respective ZAHB locations and *if* standard reddening values are assumed. Sweigart et al (1987) carried out an exhaustive examination of the models and of the relevant input physics and were unable to come up with a satisfactory explanation for the observations, unless helium is anticorrelated with metallicity (cf Sandage 1982). However, this would be completely contrary to current ideas about how chemical enrichment proceeds, as well as being in conflict with He abundance determinations from the *R*-method (e.g. Buonanno et al 1985) and (probably) from fits to the observed luminosity widths of cluster HB populations (e.g. Dorman et al 1989).

Hence, to maintain the canonical framework, suggestions were put forward that either the cluster reddening values that have generally been assumed are incorrect (e.g. Caputo 1988) or the RR Lyrae stars in the most metal-deficient clusters are highly evolved and are therefore much brighter than ZAHB stars (Lee, Demarque & Zinn 1990). But both of these alternatives seem indefensible. There is no doubt about M3 being essentially free of reddening, whereas the reddening of M15 has to be very close to 0.10 for the reason that this is required in order for the intrinsic colors of the turnoff stars in this cluster (see Durrell & Harris 1993) to be the same as those observed in M92 (Stetson & Harris 1988), which has the same age (VandenBerg, Bolte & Stetson 1990) and metallicity (Snedden et al 1991). The reddening of M92 is uncontroversial at  $E(B - V) = 0.02$  mag (see Stetson & Harris 1988). [Similar arguments have been put forward concerning M68, which belongs in the same metallicity group and which shows the same period shift relative to M3 as M15 (see Walker 1994).]

As Renzini & Fusi Pecci (1988), among others, have noted, the Lee et al (1990) explanation can hardly work in clusters that have very substantial RR Lyrae populations, such as M15. The variables should not constitute a big fraction of the total number of HB stars if the former are all in high-evolved states, where the evolutionary rates are particularly rapid. Lee (1991) has attempted to counter this argument by showing that the predicted period changes from his HB simulations agree well with those observed for the M15 RR Lyraes, though the uncertainties are large and his results are not entirely satisfactory because they fail to account for the existence of some stars whose periods are decreasing with time (see Silbermann & Smith 1995). According to the Lee et al hypothesis, all of the RR Lyrae stars should be evolving towards cooler temperatures and have periods that are increasing with time.

M68, however, poses an even greater challenge than M15, because it is much richer in RR Lyraes (see Table 2 by Carney et al 1992) and has many red HB stars (Walker 1994). There is little doubt that many of the RR Lyraes in this cluster are near the ZAHB. Furthermore, if one simply superimposes its CMD onto that for

M15 such that their respective turnoffs coincide, then one finds that their HBs also match (see McClure et al 1987); hence, the M15 variables are presumably also relatively near the ZAHB. If that is the case, which is by no means certain because even the color-magnitude data are not as secure as they should be (cf Figure 6 by Dorman et al 1991), then this difficulty for period-shift considerations would remain a problem for understanding the Oosterhoff-Arp effect. Even when a metallicity-dependent temperature shift of the instability strip is taken into account, it seems that canonical HB models can be reconciled with a steeply sloped period-metallicity relation only if the variables in the most metal-deficient clusters (such as M15) are significantly more evolved than those found in M3-like clusters, which are of intermediate metallicity (see Sandage 1993b).

Simon (1992), among others, has argued against such an evolutionary scenario, and if it were shown to be untenable, then canonical HB theory could well be called into question. In this regard, one cannot help but wonder whether there might be some connection with the inability of current models to account for either the chemical abundance trends along the RGBs of especially the most metal-poor GCs (see Section 2.2.2) or the luminosity functions of these same clusters (see Section 2.5). Certainly the HB of M15, in particular, has always been very hard to fathom (cf Crocker, Rood & O'Connell 1988). The main point to be emphasized here is that, although the precise slope of the relationship between  $\log P$  and  $[\text{Fe}/\text{H}]$  is uncertain at the  $\sim 20\%$  level, the Oosterhoff-Arp effect is beyond dispute and it must therefore be satisfactorily explained. A steeply sloped  $M_V(\text{HB})$  versus  $[\text{Fe}/\text{H}]$  relation could well be the only way to accommodate the pulsation data.

Our understanding of the Population II distance scale is clearly less than satisfactory. The nearby subdwarfs appear to define a tight main-sequence locus that can be used to derive the distance to any globular cluster with accurate photometry of its main-sequence stars and a reliable reddening estimate. When applied to M92, this approach suggests that  $M_V(\text{HB}) \approx 0^m.40$  at the metal-poor end. This distance scale is consistent with the Galactic Cepheid scale as applied to the LMC and cluster RR Lyrae stars there. It is also consistent with the (fairly model-dependent) magnitudes derived for the most metal-poor cluster RR Lyraes, with B-W results for M92 (possibly), and with the luminosities inferred from the Oosterhoff-Arp period-metallicity relation. Taking the subdwarf distance for M92 as being free of systematic errors, we find an age for M92 of  $15.8 \pm 2$  Gyr based on the Vandenberg et al (1996) models. (There is then an additional possible systematic error with the evolutionary calculations; in particular, we expect a reduction of  $\sim 1$  Gyr if uninhibited helium diffusion occurs). However, the unexplained discrepancy between this bright RR Lyrae magnitude zero-point and the fainter one derived via B-W and statistical par-

allax studies of field RR Lyrae stars leaves open the possibility that systematic errors remain in the distance scale. If the fainter scale turns out to be the correct one, then the age derived for M92 based on the same models mentioned above would be  $\sim 19$  Gyr.

As a final remark, we point out that the CMDs of metal-rich GCs—like 47 Tuc, which is the most thoroughly studied of such systems (Hesser et al 1987)—appear to pose few difficulties for canonical stellar evolutionary theory. For instance, Bell (1992) has obtained a superb match to the entire CMD of 47 Tuc brighter than the turnoff (including the RGB, the HB, and the asymptotic giant branch), using stellar models for the observed  $[\text{Fe}/\text{H}] = -0.8$  (Brown, Wallerstein & Oke 1990). His fits assumed a true distance modulus of  $m - M = 13.33$ , which is identical to that recently derived by Montegriffo et al (1995) from their extensive photometry, very similar to those estimates contained in catalogues of cluster properties (cf Webbink 1985, Djorgovski 1993), and within 0.05 mag of that adopted by Hesser et al (1987), who derived an age of 13.5 Gyr. (This age should be reduced to perhaps 12 Gyr given that the models used by Hesser et al did not allow for He diffusion or Coulomb interactions in the equation of state.) Our supposition that the most metal-poor globular clusters are the oldest ones is almost certainly correct.

#### 4. SUMMARY

The quest to determine accurate globular cluster ages and to ascertain when the first of these objects formed in the Galaxy (and how long that formation epoch lasted—see the companion review by Stetson et al 1996) is, without a doubt, one of the grand adventures in astronomy. It involves nearly all aspects of stellar astronomy and has profound importance for some of the biggest questions our species has ever asked: How did our Galaxy form? How old is the Universe? Is the Universe infinite, and will it exist forever? It has taken the effort of many researchers in many countries around the world to get to where we are now. Despite the enormous progress that has been made, the answers to such age-related questions remain elusive. Although the globular clusters are simple in many respects, being composed of low-mass stars of essentially the same age and initial chemical composition, our understanding of stellar evolution has not yet progressed far enough to be able to explain, in a fully self-consistent way and with sufficient precision, the entire wealth of information that we have garnered through the use of sophisticated observational techniques. This is particularly true for the later stages of evolution: Models for upper main-sequence and turnoff stars appear to meet the challenge of the observational tests so far devised.

As many others have found previously, our best estimate of the ages of the most metal-poor GCs, which are presumably the oldest, is  $15_{-3}^{+5}$  Gyr (allowing for the full impact of helium diffusion, which was not treated in the models that were fitted to the M92 CMD). This figure could easily be off by 1–2 Gyr in either direction, but it would be very difficult (in our opinion) to reduce it to below 12 Gyr, or to increase it much above 20 Gyr. (These can probably be regarded as  $\approx 2\sigma$  limits, though it is difficult to assign confidence intervals in this way because the errors in the models and in the various procedures used to obtain an age estimate are likely not Gaussian in sum total.) We favor an imbalance in the attached error bar for two reasons. First, the effects of He diffusion were allowed for in this estimate: Ignoring them would imply about a 7% increase in age. Second, we have opted for the distance scale defined by the local subdwarfs, which is within 0.1–0.2 mag of that implied by the calibration of RR Lyraes in the LMC (using the Cepheid-based distance to this system) and studies of the pulsational properties of cluster variable stars. The use of the distance scale based on B-W and statistical parallax measures of field RR Lyraes would also imply higher ages for the GCs. This estimate, which has remained essentially unchanged for (at least) the past 25 years despite steady refinements in both theory and observations during this period, should be regarded as quite a robust result by the cosmology community.

Although it is a common practice to simply add 1 Gyr to the best estimate of globular cluster ages to account for the formation time of these objects, there is potentially a fairly large range in the number that must be added to derive the age of the Universe. As shown in Figure 10 (for a more detailed analysis see Tayler 1986), the actual correction depends sensitively on the values of  $H_0$ ,  $\Omega_{\text{Matter}}$ , and the formation redshift of the GCs. The redshift at which galaxies like the Milky Way formed remains one of the most important open questions in observational cosmology. However, based on chemical-abundance measurements in absorption-line systems along the line of sight to distant quasars, it appears that gas in the Universe underwent significant enrichment between redshifts  $z$  of 3.5 to 2 (e.g. Lanzetta, Wolfe & Turnshek 1995, Wolfe et al 1995), and it therefore seems likely that the formation epoch of GCs was earlier than  $z = 3.5$  (also see Sandage 1993c). Although there are theoretical reasons for believing that globular clusters formed before galaxies (Peebles & Dicke 1968), perhaps at redshifts as large as 10, the existence of field halo stars in the halo of the Milky Way that are significantly more metal-poor than GC stars may argue against this hypothesis. Still, with these limits on  $z$ , the age of the Universe is very likely  $\lesssim 10^9$  yr (see Figure 10) older than the Galactic GC system.

Solutions to the field equations of General Relativity for isotropic, homogeneous universes are referred to as Friedmann, Friedmann-Lemaître, or Friedmann-Robertson-Walker models. These include, as a special case, the Einstein-de Sitter solution, in which  $\Omega_{\text{Total}} = 1$  (and the curvature of space is zero). Einstein-de Sitter universes are currently favored because  $\Omega_{\text{Total}} = 1$  appears to be a natural consequence of inflationary theory, which provides (a) a solution to the “horizon” problem posed by the smoothness of the cosmic microwave background on large scales, (b) a physical basis for the inhomogeneities that seeded galaxy formation, and (c) an explanation for the apparently very small amount of curvature in the Universe (the “flatness” problem). A choice motivated largely by elegance, and the application of Occam’s razor, is the setting of the cosmological constant ( $\Lambda$ ) to zero: The resultant matter-dominated Einstein-de Sitter model is arguably the standard model in cosmology today.

The solid curves in Figure 11 indicate loci of constant expansion age on the  $\Omega_{\text{Matter}}$  versus  $H_0$  plane for Friedmann models with  $\Lambda = 0$ . Because we believe that a firm lower limit to GC ages is 12 Gyr (equal to our best estimate minus a generous error bar of 3 Gyr), the 12 Gyr curve should be shifted to somewhat

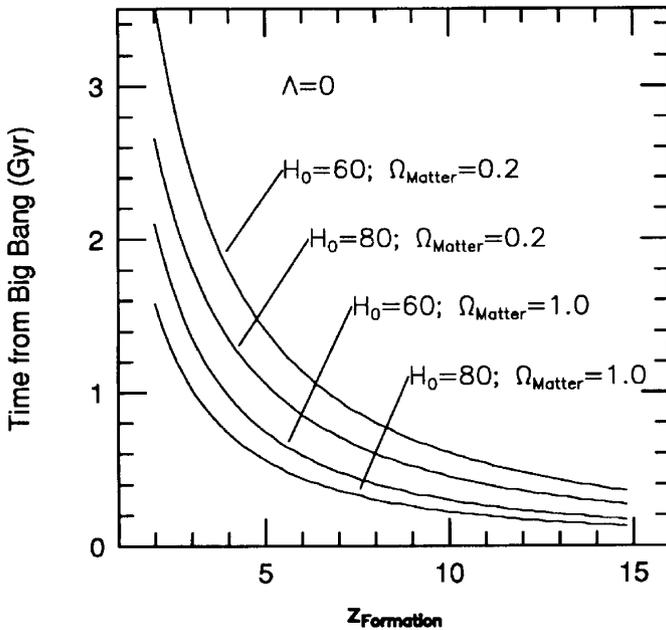


Figure 10 The time from the Big Bang to a given redshift as a function of various combinations of  $H_0$  and  $\Omega_{\text{Total}} = \Omega_{\text{Matter}}$  (i.e. the cosmological constant is assumed to be zero). Friedmann cosmological models are assumed.

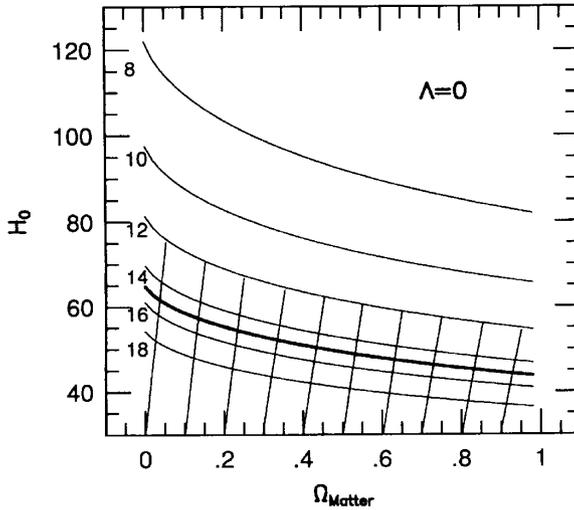


Figure 11 Expansion-age isochrones (for ages from 8 to 18 Gyr, as indicated) as a function of  $H_0$  and  $\Omega_{\text{Matter}}$ , assuming Friedman cosmological models. Given that globular clusters set a firm lower limit of 12 Gyr for the age of the Universe (see text), those combinations of  $H_0$  and  $\Omega_{\text{Matter}}$  outside of the hatched area are precluded, unless the cosmological constant is nonzero. Our best estimate of GC ages is represented by the thick 15 Gyr locus.

lower  $H_0$  values (at fixed  $\Omega_{\text{Matter}}$ ) to allow for the elapsed time between the Big Bang and GC formation (see Figure 10). But even as it stands,  $\Omega_{\text{Matter}} = 1$ ,  $\Lambda = 0$  Einstein–de Sitter universes are rejected at the 95% confidence level for  $H_0 = 65 \pm 10\% \text{ km s}^{-1} \text{ Mpc}^{-1}$ . Furthermore, if  $H_0 \sim 80 \pm 8 \text{ km s}^{-1} \text{ Mpc}^{-1}$  [see van den Bergh’s (1995) summary of the *HST*  $H_0$  Key Project results], then our age-based upper limit to  $H_0$  is inconsistent at the  $\sim 3\sigma$  level.

The two most widely discussed alternatives to the standard model to bring expansion ages into concordance with those derived for GCs are low- $\Omega_{\text{Matter}}$ ,  $\Lambda = 0$ , spatially open universes or low- $\Omega_{\text{Matter}}$ , spatially flat universes that have a nonzero value of  $\Lambda$ . For the first case, if we assume a large value for the formation redshift, then there is no  $1\sigma$  overlap between  $H_0 = 80 \pm 8 \text{ km s}^{-1} \text{ Mpc}^{-1}$  and our GC-age constraint on the Hubble constant: For  $H_0 < 70$ , the  $1\sigma$  error bars do overlap. For the second case (see the excellent review on nonzero  $\Lambda$  models by Carroll, Press & Turner 1992), a positive value of  $\Lambda$  provides a term [ $\Omega_{\Lambda} = \Lambda/(3H_0^2)$ ] that can be added to  $\Omega_{\text{Matter}}$  to give a spatially flat Universe (and preserve inflation). For instance, for  $\Omega_{\Lambda} = 0.8$ , and assuming  $\Omega_{\text{Matter}} = 0.2$ , the expansion age is 13.5 Gyr if  $H_0 = 80 \text{ km s}^{-1} \text{ Mpc}^{-1}$ . Although possibilities clearly exist for this alternative, there are already

volume- $z$  tests (e.g. the fraction of gravitationally lensed quasars; see Ostriker & Steinhardt 1995) that may exclude values for  $\Omega_{\Lambda}$  as high as this. Also, because the effects of nonzero  $\Lambda$  change with time, a whole new set of fine-tuning problems may be introduced into cosmology. The implications of stellar ages  $\sim 15$  Gyr may, indeed, become profound in the next few years as the efforts to determine  $H_0$  reduce the total (internal plus external) distance scale errors to  $\lesssim 10\%$ .

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