

# THE STANDARD COSMOLOGICAL MODEL

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**ABSTRACT.** We have a well-established standard model for cosmology and prospects for considerable additions from work in progress. I offer a list of elements of the standard model, comments on controversies in the interpretation of the evidence in support of this model, and assessments of the directions extensions of the standard model seem to be taking.

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## 1. INTRODUCTION

In the present almost frenetic rate of advance of cosmology it is useful to be reminded that the big news this year is the establishment of evidence, by two groups ([\[1\]](#), [\[2\]](#)), of detection of the relativistic curvature of the redshift-magnitude relation. The measurement was proposed in the early 1930s. Compare this to the change in the issues in particle physics since 1930. The slow evolution of cosmology has allowed ample time for us to lose sight of which elements are reasonably well established and which have been adopted by default, for lack of more reasonable-looking alternatives. Thus I think it is appropriate to devote a good part of my assigned space to a discussion of what might be included in the standard model for cosmology. I then comment on additions that may come out of work in progress.

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## 2. THE COSMOLOGICAL MODEL

Main elements of the model are easily listed: in the large-scale average the universe is close to homogeneous, and has expanded in a near homogeneous way from a denser hotter state when the 3 K cosmic background radiation was thermalized.

The standard cosmology assumes conventional physics, including general relativity theory. This yields a successful account of the origin of the light elements, at expansion factor  $z \sim 10^{10}$ . Light element formation tests the relativistic relation between expansion rate and mass density, but this is not a very searching probe. The cosmological tests discussed in [Section 3](#) could considerably improve the tests of general relativity.

The model for the light elements seems to require that the mass density in baryons is less than that needed to account for the peculiar motions of the galaxies. It is usually assumed that the remainder is nonbaryonic (or acts that way). Our reliance on hypothetical dark matter is an embarrassment; a laboratory detection would be exceedingly welcome.

In the past decade many discussions assumed the Einstein-de Sitter case, in which there are negligibly small values for the curvature of sections of constant world time and Einstein's cosmological constant  $\lambda$  (or a term in the stress-energy tensor that acts like one). This is what most of would have chosen if we were ordering. But the evidence from the relative velocities of the galaxies has long been that the mass density is less than the Einstein-de Sitter value [\[3\]](#), and other more recent observations, notably the curvature of the redshift-magnitude relation ([\[1\]](#), [\[2\]](#)), point in the same direction. Now there is increasing interest in the idea that we live in a universe in which the dominant term in the stress-energy tensor acts like a decaying cosmological constant ([\[4\]](#) - [\[10\]](#)). This is not part of the standard model, of course, but as discussed in [Section 3](#) the observations seem to be getting close to useful constraints on space curvature and  $\lambda$ .

We have good reason to think structure formation on the scale of galaxies and larger was a result of the gravitational growth of small primeval departures from homogeneity, as described by general relativity in linear perturbation theory. The adiabatic cold dark matter (ACDM) model gives a fairly definite and strikingly successful prescription for the initial conditions for this gravitational instability picture, and the ACDM model accordingly is widely used in analyses of structure formation. But we cannot count it as part of the standard model because there is at least one viable alternative, the isocurvature model mentioned in [Section 3.3](#). Observations in progress likely will eliminate at least one, perhaps establish the other as a good approximation to how the galaxies formed, or perhaps lead us to something better.

The observational basis for this stripped-down standard model is reviewed in references [\[11\]](#) and [\[12\]](#).

Here I comment on some issues now under discussion.

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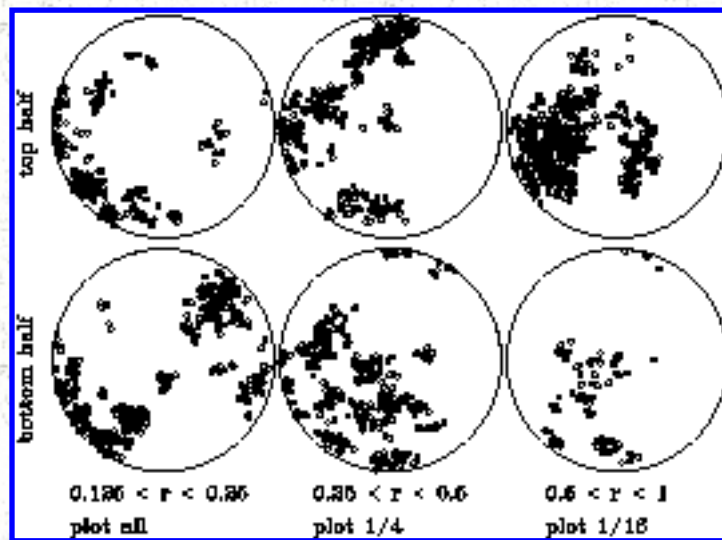
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## 2.1 The Cosmological Principle

Pietronero [13] argues that the evidence from redshift catalogs and deep galaxy counts is that the galaxy distribution is best described as a scale-invariant fractal with dimension  $D \sim 2$ . Others disagree ([14], [15]). I am heavily influenced by another line of argument: it is difficult to reconcile a fractal universe with the isotropy observed in deep surveys (examples of which are illustrated in Figs. 3.7 to 3.11 in [11] and are discussed in connection with the fractal universe in pp. 209 - 224 in [11]).



**Figure 1.** Angular distributions of particles in a realization of a fractal with dimension  $D = 2$  viewed from one of the particles in the realization. The fraction of particles plotted in each distance bin has been scaled so the expected number of particles plotted is the same in each bin.

[Fig. 1](#) shows angular positions of particles in three ranges of distance from a particle in a fractal realization with dimension  $D = 2$  in three dimensions. At  $D = 2$  the expected number of neighbors scales with distance  $R$  as  $N(< R) \propto R^2$ , and I have scaled the fraction of particles plotted as  $R^{-2}$  to get about the same number in each plot. The fractal is constructed by placing a stick of length  $L$ , placing on either end the centers of sticks of length  $L / \lambda$ , where  $\lambda = 2^{1/D}$ , with random orientation, and iterating to smaller and larger scales. The particles are placed on the ends of the shortest sticks in the clustering hierarchy. This construction with  $D = 1.23$  (and some adjustments to fit the galaxy three- and four-point correlation functions) gives a good description of the small-scale galaxy clustering [16]. The fractal in [Fig. 1](#), with  $D = 2$ , the dimension Pietronero proposes, does not look at all like deep sky maps of galaxy distributions, which show an approach to isotropy with increasing depth. This cannot happen in a scale-invariant fractal: it has no characteristic length.

A characteristic clustering length for galaxies may be expressed in terms of the dimensionless two-point correlation function defined by the joint probability of finding galaxies centered in the volume elements  $dV_1$  and  $dV_2$  at separation  $r$ ,

$$dP = n^2[1 + \xi_{gg}(r)]dV_1dV_2. \quad (1)$$

The galaxy two-point function is quite close to a power law,

$$\xi = (r_0/r)^\gamma, \quad \gamma = 1.77, \quad 10 \text{ kpc} \lesssim hr \lesssim 10 \text{ Mpc}, \quad (2)$$

where the clustering length is

$$r_0 = 4.5 \pm 0.5 \text{ Mpc}, \quad (3)$$

and the Hubble parameter is

$$H_0 = 100h \text{ km s}^{-1} \text{ Mpc}^{-1}. \quad (4)$$

The rms fluctuation in galaxy counts in a randomly placed sphere is  $\delta N/N = 1$  at sphere radius  $r = 1.4r_0 \sim 6h^{-1} \text{ Mpc}$ , to be compared to the Hubble distance (at which the recession velocity approaches the velocity of light),  $cH_0^{-1} = 3000h^{-1} \text{ Mpc}$ .

The isotropy observed in deep sky maps is consistent with a universe that is inhomogeneous but spherically symmetric about our position. There are tests, as discussed by Paczynski and Piran [17]. For example, we have a successful theory for the origin of the light elements as remnants of the expansion and cooling of the universe through  $kT \sim 1 \text{ MeV}$  [18]. If there were a strong radial matter density gradient out to the Hubble length we could be using the wrong local entropy per baryon, based on conditions at the Hubble length where the CBR came from, yet the theory seems to be successful. But to most people the compelling argument is that distant galaxies look like equally good homes for observers like us: it would be startling if we lived in one of the very few close to the center of symmetry.

Mandelbrot [19] points out that other fractal constructions could do better than the one in Fig. 1. His example does have more particles in the voids defined by the strongest concentrations in the sky, but it seems to me to share the distinctly clumpy character of Fig. 1. It would be interesting to see a statistical test. A common one expands the angular distribution in a given range of distances in spherical harmonics,

$$a_l^m = \int d\Omega \sigma(\Omega) Y_l^m(\Omega), \quad (5)$$

where  $\sigma$  is the surface mass density as a function of direction  $\omega$  in the sky. The integral becomes a sum if the fractal is represented as a set of particles. A measure of the angular fluctuations is

$$e_l = l \sum_{-l < m < l} |a_l^m|^2 / (a_0^0)^2, \quad (6)$$

where

$$\langle \sigma^2 \rangle / \langle \sigma \rangle^2 - 1 = \sum_{l \geq 1} e_l / l. \quad (7)$$

In the approximation of the sum as an integral  $e_l$  is the contribution to the variance of the angular distribution per logarithmic interval of  $l$ . It will be recalled that the zeros of the real and imaginary parts of  $Y_l^m$  are at separation  $\theta = \pi / l$  in the shorter direction, except where the zeros crowd together near the poles and  $Y_l^m$  is close to zero. Thus  $e_l$  is the variance of the fractional fluctuation in density across the sky on the angular scale  $\theta \sim \pi / l$  and in the chosen range of distances from the observer.

I can think of two ways to define the dimension of a fractal that produces a close to isotropic sky. First, each octant of a full sky sample has half the diameter of the full sample, so one might define  $D$  by the fractional departure of the mean density within each octant from the mean in the full sample,

$$(e_2)^{1/2} \sim 2^{3-D} - 1. \quad (8)$$

Thus in [Fig. 1](#), with  $D = 2$ , the quadrupole anisotropy  $e_2$  is on the order of unity. Second, one can see the idea that the mean particle density varies with distance  $r$  from a particle as  $r^{-(3-D)}$ . Then the small angle (large  $l$ ) Limber approximation to the angular correlation function  $w(\theta)$  is [\[20\]](#)

$$1 + w(\theta = \pi/l) \sim \int^l e_l dl / l \propto \int_0^1 du [u^2 + (\pi/l)^2]^{-(3-D)/2}. \quad (9)$$

To find  $e_l$  differentiate with respect to  $l$ . At  $D = 2$  this gives  $e_l \sim 1$ : the surface density fluctuations are independent of scale. At  $0 < 3 - D \ll 1$ ,  $e_l \sim (3 - D) / l$ . The X-ray background fluctuates by about  $\delta f / f \sim 0.05$  at  $\theta = 5^\circ$ , or  $l \sim 30$ . This is equivalent to  $D \sim 3 - l (\delta f / f)^2 \sim 2.9$  in the fractal model in Eq. (9).

The universe is not exactly homogeneous, but it seems to be remarkably close to it on the scale of the

Hubble length. It would be interesting to know whether there is a fractal construction that allows a significantly larger value of  $3 - D$  for given  $e_l$  than in this calculation.

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## 2.2 The Hubble Redshift-Distance Relation

Expansion that preserves homogeneity requires that the mean rate of change of separation of pairs of galaxies with separation  $R$  varies as the Hubble law,

$$v = HR. \quad (10)$$

The redshift-distance relation for type Ia supernovae gives an elegant demonstration of this relation ([1], [2]). Arp ([21], [22]) points out that such precision tests do not directly apply to the quasars, and he finds fascinating evidence in sky maps for associations of quasars with galaxies at distinctly lower redshifts. But there is a counterargument, along lines pioneered by Bergeron [23], as follows.

A quasar spectrum may contain absorption lines characteristic of a cloud of neutral atomic hydrogen at surface density  $\sigma_{\text{HI}} \gtrsim 3 \times 10^{17}$  atoms  $\text{cm}^{-2}$ . If this absorption system is at redshift  $z \lesssim 1$  a galaxy at the same redshift is close enough that there is a reasonable chance observing it, and with high probability an optical image does show a galaxy close to the quasar and at the redshift of the absorption lines ([24], [25]). Also, when a galaxy image appears in the sky close to a quasar at higher redshift then with high probability the quasar spectrum has absorption lines at the redshift of the galaxy. We have good evidence the galaxy is at the distance indicated by its redshift. We can be sure the quasar is behind the galaxy: the quasar light had to have passed through the galaxy to have produced the absorption lines. If quasars were not at their cosmological distances we ought to have examples of a quasar appearing close to the line of sight to a lower redshift galaxy and without the characteristic absorption lines produced by the gas in and around the galaxy.

Arp's approach to this issue is important, but I am influenced by what seems to be this direct and clear interpretation of the Bergeron effect, that indicates redshift is a good measure of distance for quasars as well as galaxies.

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## 2.3 The Expansion of the Universe

In the relativistic Friedmann-Lemaître cosmological model the wavelength of a freely propagating photon is stretched in proportion to the expansion factor from the epoch of emission to detection:

$$1 + z = \frac{\lambda_{\text{obs}}}{\lambda_{\text{em}}} = \frac{a_{\text{obs}}}{a_{\text{em}}}. \quad (11)$$

The first expression defines the redshift  $z$  in terms of the ratio of observed wavelength to wavelength at emission. The cosmological expansion parameter  $a(t)$  is proportional to the mean distance between conserved particles.

The most direct evidence that the redshift is a result of expansion is the thermal spectrum of the CBR [26]. In a tired light model in a static universe the photons suffer a redshift that is proportional to the distance travelled, but in the absence of absorption or emission the photon number density remains constant. In this case a significant redshift makes an initially thermal spectrum distinctly not thermal and inconsistent with the measured CBR spectrum. One could avoid this by assuming the mean free path for absorption and emission of CBR photons is much shorter than the Hubble length, so relaxation to thermal equilibrium is much faster than the rate of distortion of the spectrum by the redshift. But this opaque universe is quite inconsistent with the observation of radio galaxies at redshifts  $z \sim 3$  at CBR wavelengths. That is, the universe cannot have an optical depth large enough to preserve a thermal CBR spectrum in a tired light model. In the standard world model the expansion has two effects: it redshifts the photons, as  $\lambda \propto a(t)$ , and it dilutes the photon number density, as  $n \propto a(t)^{-3}$ . The result is to cool the CBR while keeping its spectrum thermal. Thus the expanding universe allows a self-consistent picture: the CBR was thermalized in the past, at a time when when the universe was denser, hotter, and optically thick.

I have not encountered any serious objection to this argument; the issue is the expansion factor. In the relativistic Friedmann-Lemaître model the expansion of the universe traces back at least as far as redshift  $z \sim 10^{10}$ , when the light elements formed in observationally reasonable amounts [18]. In the model of Arp et al. [27] the expansion and cooling traces back to a redshift only moderately greater than the largest observed values,  $z \sim 5$ , when there would have been a burst of creation of matter and radiation followed by rapid clearing of the dust that thermalized the radiation. The Arp et al. picture for the origin of the light elements has not been widely debated. If it were agreed that it is viable then a choice between this and the Friedmann-Lemaître model would depend on other tests, such as the angular fluctuations in the CBR, as discussed next.

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### 3. THE COSMOLOGICAL TESTS

The tests in [Table 1](#) are organized in four categories: spacetime geometry, galaxy peculiar velocities, structure formation, and early universe physics. I offer grades for three sets of parameter choices. As the tests improve we may learn that one narrowly constrained set of values of the cosmological parameters receives consistent passing grades, or else that we have to cast our theoretical net more broadly.

**Table 1: The cosmological tests.**

Test	Einstein- de Sitter	$\Omega = 0.25 \pm 0.1$	
		Flat	Open
1a. World time $t(z)$ : ages of stars & elements	<b>X??</b>	. $\checkmark$	. $\checkmark$
1b. SNeIa Redshift- magnitude relation	<b>X</b>	. $\checkmark$	. <b>X??</b>
1c. Counts: $dN =$ $f(m, z) dm dz$	?	. ?	. ?
1d. Lensing of quasars by galaxies	$\checkmark$	. <b>X??</b>	. $\checkmark$
2a. Dynamics on scales $\lesssim 10$ Mpc	<b>X</b>	. $\checkmark$	. $\checkmark$
2b. Dynamics on larger scales	<b>X?</b>	. $\checkmark$	. $\checkmark$
3a. 2 <sup>nd</sup> moments: CBR & galaxy distributions	<b>X??</b>	. $\checkmark^{??}$	. $\checkmark^{??}$
3b. Cluster mass function & evolution	<b>X?</b>	. $\checkmark$	. $\checkmark$
3c. Baryon mass fraction in clusters	<b>X?</b>	. $\checkmark$	. $\checkmark$
3d. Galaxy formation	?	. ?	. ?
4. Fundamental physics	?	. ?	. ?

$\checkmark$  = conditional pass;      **X** = conditional fail.

**✓ = conditional pass;    ✗ = conditional fail.**

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### 3.1 Spacetime Geometry

In the relativistic Friedmann-Lemaître cosmological model the mean spacetime geometry (ignoring curvature fluctuations produced by local mass concentrations in galaxies and systems of galaxies) may be represented by the line element

$$ds^2 = dt^2 - a(t)^2 \left[ \frac{dr^2}{1 \pm r^2/R^2} + r^2(d\theta^2 + \sin^2 \theta d\phi^2) \right], \quad (12)$$

where the expansion rate satisfies the equation

$$H^2 = \left( \frac{\dot{a}}{a} \right)^2 = \frac{8}{3} \pi G \rho \pm \frac{1}{a^2 R^2} + \frac{\Lambda}{3}, \quad (13)$$

which might be approximated as

$$H^2 = H_0^2 [\Omega(1+z)^3 + \kappa(1+z)^2 + \lambda]. \quad (14)$$

The last equation defines the fractional contributions to the square of the present Hubble parameter  $H_0$  by matter, space curvature, and the cosmological constant (or a term in the stress-energy tensor that acts like one). The time-dependence assumes pressureless matter and constant  $\lambda$ . Other notations are in the literature; a common practice in the particle physics community to add the matter and  $\lambda$  terms in a new density parameter,  $\omega' = \omega + \lambda$ . I prefer keeping them separate, because the observational signatures of  $\omega$  and  $\lambda$  can be quite different.

By 1930 people understood how one would test the space-time geometry in these equations, and as I mentioned there is at last direct evidence for the detection of one of the effects, the curvature of the relation between redshift and apparent magnitude ([1], [2]). As indicated in line 1b, the measured curvature is inconsistent with the Einstein-de Sitter model in which  $\omega = 1$  and  $\lambda = 0 = \kappa$ . The measurements also disagree with a low density model with  $\lambda = 0$ , though the size of the discrepancy approaches the size of the error flags, so I assign a weaker failing grade for this case. The measurements are magnificent. The issue yet to be thoroughly debated is whether the type Ia supernovae observed at redshifts  $0.5 \lesssim z \lesssim 1$  are drawn from essentially the same population as the nearer ones.

In a previous volume in this series Krauss [28] discusses the time-scale issue. Stellar evolution ages and radioactive decay ages do not rule out the Einstein-de Sitter model, within the still considerable

uncertainties in the measurements, but the longer expansion time scales of the low  $\omega$  models certainly relieve the problem of interpretation of the measurements. Thus I enter a tentative negative grade for the Einstein-de Sitter model in line 1a.

In the analysis by Falco et al. [29] of the rate of lensing of quasars by foreground galaxies (line 1d) for a combined sample of lensing events detected in the optical and radio, the  $2\sigma$  bound on the density parameter in a cosmologically flat ( $\kappa = 0$ ) universe is  $\omega > 0.38$ . The SNeIa redshift-magnitude relation seems best fit by  $\omega = 0.25$ ,  $\lambda = 0.75$ , a possibly significant discrepancy. A serious uncertainty in the analysis of the lensing rate is the number density of early-type galaxies in the high surface density branch of the fundamental plane at luminosities  $L \sim L_*$ , the luminosity of the Milky Way. If further tests confirm an inconsistency of the lensing rate and the redshift-magnitude relation the lesson may be that  $\lambda$  is dynamical, rolling to zero, as Ratra & Quillen [30] point out.

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### 3.2 Biasing and Large-Scale Velocities

The relation between the mass density parameter  $\omega$  and the gravitational motions of the galaxies is an issue rich enough for a separate category in [Table 1](#). It has been known for the past decade that if galaxies were fair tracers of mass then the small-scale relative velocities of the galaxies would imply that  $\omega$  is well below unity [\[3\]](#). If the mass distribution were smoother than that of the galaxies, the smaller mass fluctuations would require a larger mean mass density to gravitationally produce the observed galaxy velocities. Davis, Efstathiou, Frenk & White [\[31\]](#) were the first to show that such a biased distribution of galaxies relative to mass readily follows in numerical N-body simulations of the growth of structure, and the demonstration has been repeated in considerable detail ([\[32\]](#), [\[33\]](#), and references therein). This is a serious argument for the biasing effect. But here are three arguments for the proposition that galaxies are fair tracers of mass for the purpose of estimating  $\omega$ .

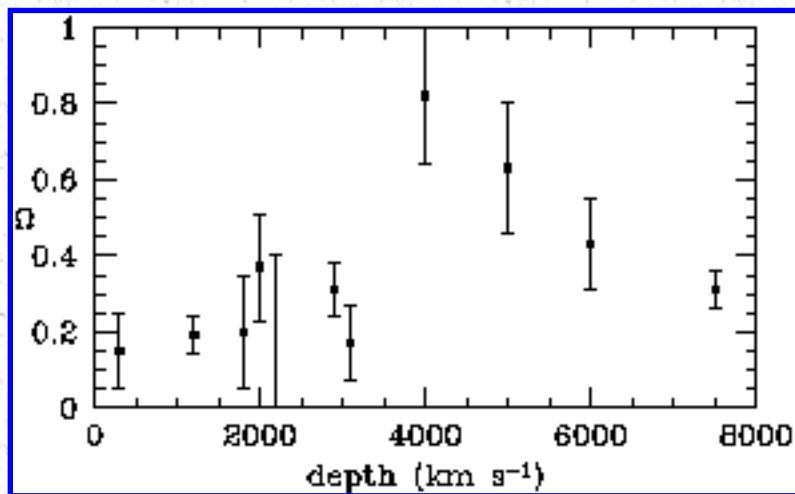
First, in many numerical simulations dwarf galaxies are less strongly clustered than giants. This is reasonable, for if much of the mass were in the voids defined by the giant galaxies, as required if  $\omega = 1$ , then surely there would be remnants of the suppressed galaxy formation in the voids, irregular galaxies that bear the stigmata of a hostile early environment. The first systematic redshift survey showed that the distributions of low and high luminosity galaxies are strikingly similar [\[34\]](#). No survey since, in 21-cm, infrared, ultraviolet, or low surface brightness optical, has revealed a void population. There is a straightforward interpretation: the voids are nearly empty because they contain little mass.

Second, one can use the galaxy two-point correlation function in Eq. (1) and the mass autocorrelation function  $\xi_{\rho\rho}$  from a numerical simulation of structure formation to define the bias function

$$b(r, t) = [\xi_{gg}(r, t)/\xi_{\rho\rho}(r, t)]^{1/2}. \quad (15)$$

In numerical simulations  $b$  typically varies quite significantly with separation and redshift [\[32\]](#). That is, the galaxies give a biased representation of the statistical character of the mass distribution in a typical numerical simulation. The issue is whether the galaxies, or the models, or both, are biased representations of the statistical character of the real mass distribution. What particularly strikes me is the observation that the low order galaxy correlation functions have some simple properties. The galaxy two-point function is close to a power law over some three orders of magnitude in separation (Eq. 2). The value of the power law index  $\gamma$  changes little back to redshift  $z \sim 1$ . Within the clustering length  $r_0$  the higher order correlation functions are consistent with a power law fractal. A reasonable presumption is that the regularity exhibited by the galaxies reflects a like regularity in the mass, because galaxies trace mass. I am impressed by the power of the numerical simulations, and believe they reflect important aspects of reality, but do not think we should be surprised if they do not fully represent other aspects,

such as relatively fine details of the mass distribution.



**Figure 2.** The density parameter derived from galaxy peculiar motions on the assumption galaxies trace mass. From the left, the estimates are based on the Local Group of galaxies, clusters of galaxies, the peculiar infall toward the [Virgo Cluster \[35\]](#), and the analyses in references [\[36\]](#) to [\[38\]](#).

The third argument deals with the idea that blast waves or radiation from the formation of a galaxy may have affected the formation of nearby galaxies, producing scale-dependent bias. In this case the apparent value of the density parameter derived from gravitational motions within systems of galaxies on the assumption galaxies trace mass would be expected to vary with increasing scale, approaching the true value when derived from relative motions on scales larger than the range of influence of a forming galaxy. [Fig. 2](#) shows a test. The abscissa at the entry for clusters of galaxies is the comoving radius of a sphere that contains the mass within the Abell radius. The estimates at larger scales are plotted at approximate values of the radius of the sample. If it were not for the last two points at the right-hand side of [Fig. 2](#), one might conclude that the apparent density parameter is increasing to the true value  $\Omega \sim 1$  at  $R \sim 50h^{-1}$  Mpc. But considering the last two points, and the sizes of the error flags, it is difficult to see any evidence for scale-dependent bias.

I assign a strongly negative grade for the Einstein-de Sitter model in line 2a in [Table 1](#), based on galaxy motions on relatively small scales, because biasing certainly is required if  $\Omega = 1$  and I have argued there is no evidence for it. The more tentative grade in line 2b is based on [Fig. 2](#): the apparent value of the density parameter does not seem to scale with depth.

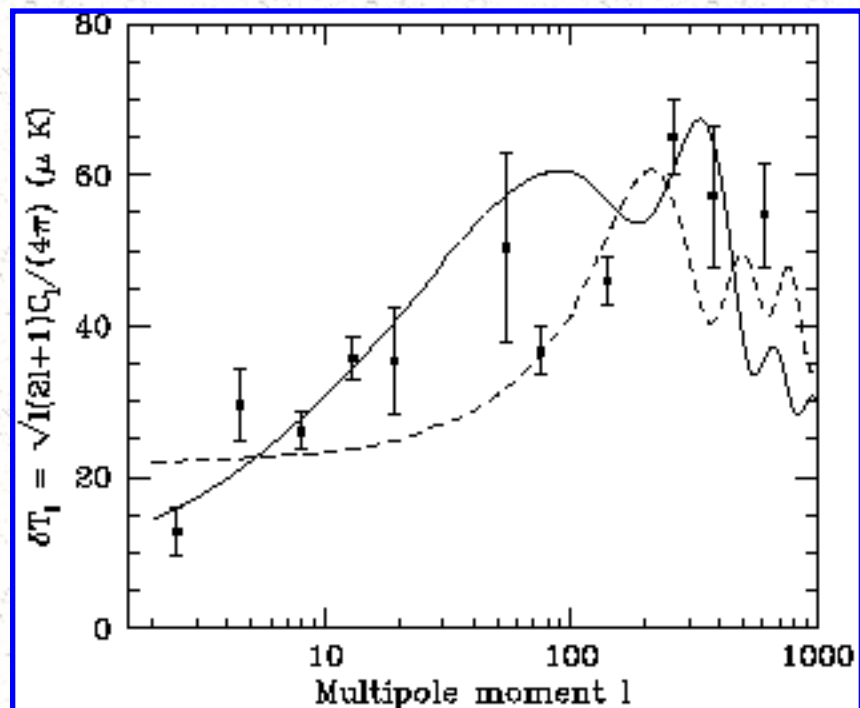
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### 3.3 Structure Formation

The Friedmann-Lemaître model is unstable to the gravitational growth of departures from a homogeneous mass distribution. The present large-scale homogeneity could have grown out of primeval chaos, but the initial conditions would be absurdly special. That is, the Friedmann-Lemaître model requires that the present structure - the clustering of mass in galaxies and systems of galaxies - grew out of small primeval departures from homogeneity. The consistency test for an acceptable set of cosmological parameters is that one has to be able to assign a physically sensible initial condition that evolves into the present structure of the universe. The constraint from this consideration in line 3c is discussed by White et al. [44], and in line 3b by Bahcall et al. ([45], [46]). Here I explain the cautious ratings in line 3a.

As has been widely discussed, it may be possible to read the values of  $\omega$  and other cosmological parameters from the spectrum of angular fluctuations of the CBR ([47] and references therein). This assumes Nature has kept the evolution of the early universe simple, however, and we have hit on the right picture for its evolution. We may know in the next few years. If the precision measurements of the CBR anisotropy from the MAP and PLANCK satellites match in all detail the prediction of one of the structure formation models now under discussion it will compel acceptance. But meanwhile we should bear in mind the possibility that Nature was not kind enough to have presented us with a simple problem.



**Figure 3.** Angular fluctuations of the CBR in low density cosmologically flat adiabatic (dashed line) and isocurvature (solid line) CDM models for structure formation. The variance of the CBR temperature anisotropy per logarithmic interval of angular scale  $\theta = \pi / l$  is  $(T_l)^2$ , as in Eqs. (5) to (7). Data are from the compilation by Ratra [48].

An example of the possible ambiguity in the interpretation of the present anisotropy measurements is shown in Fig. 3. The two models assume the same dynamical actors - cold dark matter (CDM), baryons, three families of massless neutrinos, and the CBR - but different initial conditions. In the adiabatic model the primeval entropy per conserved particle number is homogeneous, the space distribution of the primeval mass density fluctuations is a stationary random process with the scale-invariant spectrum  $\propto k$ , and the cosmological parameters are  $\omega = 0.35$ ,  $\lambda = 0.65$ , and  $h = 0.625$  (following [49]). The isocurvature initial condition in the other model is that the primeval mass distribution is homogeneous - there are no curvature fluctuations - and structure formation is seeded by an inhomogeneous composition. In the model shown here the primeval entropy per baryon is homogeneous, to agree with the standard model for light element production, and the primeval distribution of the CDM has fluctuation spectrum

$$P(k) \propto k^m, \quad m = -1.8. \quad (16)$$

The cosmological parameters are  $\omega = 0.2$ ,  $\lambda = 0.8$ , and  $h = 0.7$ . The lower density parameter produces a more reasonable-looking cluster mass function for the isocurvature initial condition [50]. In both models the density parameter in baryons is  $\omega_B = 0.03$ , the rest of  $\omega$  is in CDM, and space sections are flat ( $\lambda = 1 - \omega$ ). Both models are normalized to the large-scale galaxy distribution. The adiabatic initial condition follows naturally from inflation, as a remnant of the squeezed field that drove the rapid expansion. A model for the isocurvature condition assumes the CDM is (or is the remnant of) a massive scalar field that was in the ground level during inflation and became squeezed to a classical realization. In the simplest models for inflation this produces  $m = -3$  in Eq. (16). The tilt to  $m = -1.8$  requires only modest theoretical ingenuity [51]. That is, both models have pedigrees from commonly discussed early universe physics.

The lesson from Fig. 3 is that at least two families of models, with different relations between  $\omega$  and the value of  $l$  at the peak, come close to the measurements of the CBR fluctuation spectrum, within the still substantial uncertainties. An estimate of  $\omega$  from the CBR anisotropy measurements thus may depend on the choice of the model for structure formation. Programs of measurement of  $\delta T_l$  in progress should be capable of distinguishing between the adiabatic and isocurvature models, even given the freedom to adjust the shape of  $P(k)$ . The interesting possibility is that some other model for structure formation with a very different value of  $\omega$  may give an even better fit to the improved measurements.

I assign a failing grade to the Einstein-de Sitter model in line 3a because the adiabatic and isocurvature

models both prefer low  $\omega$  ([52], [53]). I add question marks to indicate this still is a model-dependent result.

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### 3.4 Constraints From Fundamental Physics

In their version of [Table 1](#) Dekel, Burstein, & White [54] give the Einstein-de Sitter model the highest grade on theoretical grounds, and a cosmologically flat model with  $\lambda$  the next highest grade. The point is well taken: this is the order most of us would choose. The issue is whether Nature agrees with our ideas of elegance, or maybe prefers physics that produces an open universe ([\[55\]](#) - [\[58\]](#)). Full closure of cosmology may come with the discovery of physics that predicts the values of  $\lambda$  and space curvature (Eq. [14]) in terms of the expansion age of the universe, consistent with all the other constraints in [Table 1](#). But since we seem to be far from that goal I am inclined to omit entries in line 4.

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## 4. Concluding Remarks

We have a secure if still schematic standard model for cosmology, and the prospect for considerable enlargement from the application of the cosmological tests. The theoretical basis for the tests was discovered seven decades ago. A significant application likely will take a lot less than seven more decades: the constraints in [Table 1](#) already are serious, if debatable, and people know how to do better.

Application of the tests could yield a set of tightly constrained values of the cosmological parameters and a clear characterization of the primeval departure from homogeneity. If so cosmology could divide at a fixed point, the situation at  $z = 10^{15}$ , say, when the universe is well described by a slightly perturbed Friedmann-Lemaître model. One branch of research would analyze evolution from these initial conditions to the present complex structure of the universe. The other would search for the physics of the very early universe that produced these initial conditions. But before making any long-term plans based on this scenario I would wait to see whether the evidence really is that the early universe is simple enough to allow such a division of labor.

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