The Large Scale Structure of the Universe

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Abstract

Reality is a question of perspective; the further you get from the past the more concrete and plausible it seems — but as you approach the present, it inevitably seems more and more incredible.

Midnight's Children (All-India Radio)
Salman Rushdie

This is an interesting time for cosmology since it seems that our theories for the large scale structure of the Universe are being seriously challenged by the growing amount of data. The goal of these lectures is to introduce and explain the current debate on this issue. By its very nature the problem of the large scale structure is one that mixes both observational and the theoretical aspects of cosmology. The plan of the lectures is first to discuss the general theoretical framework for cosmology, and then go on to discuss the impact of key data. In this article, I shall try to build up the essential concepts from simple starting points. More details can be found in a number of books and review articles.

There are many fine books on cosmology. There are the two great "classics" *Physical Cosmology* by Peebles (1971) and *Gravitation and Cosmology* by Weinberg (1972). The 1980's brought *The Large Scale Structure of the Universe* by Peebles (1980, referred to hereafter as "LSSU"), volume 2 of *Relativistic Astrophysics* by Zel'dovich and Novikov (1980), and *The Isotropic Universe* by Raine (1981). These are becoming somewhat outdated owing to the rapid progress in cosmology in recent years, but they do discuss the fundamentals of the subject. At a pedagogical level, the book by Berry (1976), *Principles of Cosmology and Gravitation* is highly recommended.

In recent years we have *The Early Universe, Facts and Fiction* by Borner (1988), *The Early Universe* by Kolb and Turner (1990), and *Physics of the Early Universe* edited by Peacock, Heavens, and Davies (1990). This last book contains a fine review article by White (1990) on Physical Cosmology and another on inhomogeneities in the universe and in particular their contribution to microwave background anisotropy, by Efstathiou (1990).

There are numerous conference proceedings and reviews on the subject. An outstanding conference book is the "Vatican Study Week: Large Scale Motions in the Universe" (Rubin and Coyne, 1988). There is also a recent review on the large scale structure by Kashlinsky and Jones (1991).

It is of course impossible to discuss everything about the large scale structure of the universe, and even less possible to cite all the appropriate references. My strategy has therefore been to provide a background against which some of the major issues can be addressed, and to give enough recent references to enable the reader to get into the bibliography on those issues.

I have tried to stick to the question of the large scale structure and not get drawn into issues of galaxy formation or even galaxy cluster formation. There is no doubt that these have an important bearing on large scale structure, but they have not as yet played a decisive role. For example, the question of the biasing of galaxy formation has so far been tackled in a rather simplistic way and there may now be a need to go into the details of the process by looking more carefully at the galaxy formation process. On the other hand I have gone a little way into the question of inflationary universes and the origin of the fluctuation spectrum since this is a major issue as regards the scale of the largest structures.

Throughout this article I shall use a distance scale corresponding to a present value of the Hubble constant $H_0 = 100h$ km s^{-1} Mpc^{-1} and the reader can substitute her/his own favourite value for h. I shall endeavour to use "Universe" whenever I mean the place where we live, and "universe" for a model of the Universe. Similarly, "the Galaxy" is the "galaxy" where we are situated. I shall try to avoid abbreviations, but the following bits of jargon are frequently encountered and may slip into the text: "CDM" for "Cold Dark Matter", "HDM" for "Hot Dark Matter", "LSSU" for Peebles' book "The Large Scale Structure of the Universe", "MWB" for "Microwave Background" and "CBR" for the "Cosmic Background Radiation". Generally MWB and CBR are used interchangeably, despite the fact that there are many background radiations that are not in the microwave band (such as the X-ray background, but that would be an "XRB"!). I will studiously avoid "GA" for "Great Attractor" and but I will use the shortened form "S^7" for the names of the people involved in its discovery (Burstein, Davies, Dressler, Faber, Lynden-Bell, Terlevich and Wegner, in alphabetic order).

1 OVERVIEW

In this section, I shall give a brief overview of the properties of the Universe at large. On the largest scales it can be reasonably approximated as a homogeneous and isotropic medium in a state of uniform expansion and the equations can easily be written down. We find that such a simple model Universe can be described in terms of a few parameters, the expansion rate, the density, and perhaps the cosmological constant. Classical cosmology focusses on determining these by direct observation of the large scale distribution of galaxies. There are, however, many new techniques available for getting these parameters though studying the inhomogeneity of the Universe. These will be the subject of the following sections where many of the issues raised here will be discussed at greater length.

1.1 The Universe at Large

Hubble discovered the expansion of the Universe by plotting, for a sample of galaxies, the radial velocity of each galaxy as indicated by the redshift of its spectral lines against its apparent brightness. The fainter (and presumably more distant) galaxies had the greater recession velocities (or "redshifts"). If the distance to a galaxy was D Megaparsecs, and its radial velocity was V km s^{-1}, then Hubble's relationship could be expressed as

$$V = H_0 D, \tag{1}$$

where H_0 is a constant (the *Hubble constant*) measured here in units of km s^{-1} Mpc^{-1} Implicit in the relationship is the assumption that we can calibrate the distance scale by virtue of which the apparent brightness of a galaxy can be turned into a distance.

The radial component of the velocity of a galaxy relative to the observer is inferred by observing the wavelength λ_O of spectral lines that would in the laboratory have been emitted wavelength λ_E. The difference $\delta\lambda = \lambda_O - \lambda_E$ is interpreted as being due to the Doppler shift caused by the fact that the galaxy was moving at velocity

$$v = c\frac{\delta\lambda}{\lambda_E}, \tag{2}$$

relative to the observer. (We shall henceforth drop the 'E' suffix on the emitted wavelength). The redshift of the galaxy (in fact the redshift of the spectra lines) is defined as

$$z = \frac{v}{c} = \frac{\delta\lambda}{\lambda}. \tag{3}$$

Hubble's redshift-distance relation (the "Hubble Law") later became a way of estimating the distances to galaxies simply by measuring their radial velocities $D_z = H_0^{-1} cz$. (D_z has the subscript z to denote the nature of this distance estimate and to distinguish it from the true distance. We shall see later that part of the velocity cz may be due to the random motions of galaxies relative to the general cosmic expansion.)

Looked at in its most simple terms, Hubble's discovery implies that the Universe was born a finite time in our past and emerged from a state of infinite density. The subsequent discovery by Penzias and Wilson (1965) of a cosmic microwave background radiation field and its interpretation as the relict of an expansion from a hot singular state by Dicke, Peebles, Roll and Wilkinson (1965) established a definitive view of our Universe. Cosmology properly became a branch of physics, and the Hot Big Bang theory has become a paradigm of modern science.

1.1.1 Homogeneity and Isotropy

On the smallest scales the Universe contains stars that are grouped into galaxies, that are themselves grouped into clusters. Going to larger scales we have evidence for clusters of galaxy clusters, and beyond that for large scale structures ("walls" of galaxies!) extending over many tens or even hundreds of megaparsecs. Indeed pictures of the three dimensional distribution of galaxies look very inhomogeneous even on scales as large as 100 Mpc, or more. However, one should not be mislead by visual appearances. As will be explained later, this large scale inhomogeneity has rather a small amplitude in the sense that it would hardly be noticeable if the distribution of galaxies were smoothed over such large volumes. There is a clear tendency for the Universe to become more homogeneous on ever large scales.

Hubble himself commented on the remarkable large-scale isotropy of the Universe as judged from the distribution of galaxies on the sky. Today we have catalogues of galaxies penetrating to great distances (Maddox et al., 1990) and these demonstrate the isotropy of the galaxy distribution very clearly. The isotropy of the Universe is best measured through the isotropy of the cosmic microwave background radiation.

The large scale homogeneity of the Universe is more difficult to establish directly. It would seem reasonable to use the argument that we are not at the center of the Universe, so the isotropy must imply spatial homogeneity, but this is not a proof of homogeneity. The same deep galaxy catalogues provide a test of homogeneity because we can ask the question "is the Universe, sampled at various depths within this catalogue, the same?". Again the Maddox et al. (1990) catalogue provides an answer, though the method is not as simple as observing isotropy. Maddox et al. compute the galaxy clustering correlation function at various depths in their catalogue and find that the functions in the various samples scale in accordance with the hypothesis of homogeneity. Their analysis in fact goes even further than merely saying that the Universe is globally homogeneous. It has the additional implication that the deviation from homogeneity (as evidenced by the galaxy clustering) is itself the same in all their samples.

Such arguments provide compelling evidence that the Universe is not a hierarchy of the kind originally envisaged by Charlier (1908, 1922), and taken up more recently in the context of fractal distributions of galaxies by Mandelbrot (1983), Coleman, Pietronero and Sanders (1988) and others.

1.1.2 Scale Factors, Redshifts and all that

For most of what concerns us in these lectures it is sufficient to consider the Universe to be, in a first approximation, a homogeneous and isotropic distribution of particles (galaxies) that interact only through their mutual gravitational interactions. This means that we ignore any pressure contribution from their random motions, or from other components of matter. This enables us to greatly simplify the dynamical equations for the evolution of the Universe.

Consider the motion of a galaxy in the Universe that today (t_0) is at distance l_0 from us and that at time t was at a distance $l(t)$. It is convenient to define the *scale factor* $a(t)$ by

$$a(t) = \frac{l(t)}{l_0}. \tag{4}$$

Since the Universe is presumed homogeneous and isotropic, then $a(t)$ depends on neither position nor direction. It merely describes how relative the distances change as the Universe expands. We have normalized all lengths relative to their present day value and so the present value of $a(t)$ is $a(t_0) = 1$.

The Einstein equations (or their Newtonian equivalent) in the simple case of homogeneous and isotropic dust models give the differential equation for the scale factor in terms of the total mass density ρ:

$$\frac{1}{a}\frac{d^2a}{dt^2} = -\frac{4\pi G}{3}\rho, \tag{5}$$

This is supplemented by an equation expressing the conservation of matter:

$$\frac{d\rho}{dt} + 3\frac{\dot{a}}{a}\rho = 0, \tag{6}$$

which is equivalent to

$$\rho(t) = \rho_0 a^{-3}. \tag{7}$$

Note that (5) is not valid if there is any substantial pressure due to the matter in the universe, and in that case we also need to modify (6). We shall make these modifications at a later time when needed, for the moment we are only discussing the Universe at the present time and in its recent past when equations (5, 6, 7) are thought to be a good approximation.

The *Hubble Parameter* is defined as

$$H = \frac{\dot{a}}{a} = \frac{\dot{l}}{l}, \tag{8}$$

and is a function of time. H describes the rate of expansion of the Universe and has units of inverse time. It is experimentally measured as a velocity increment per unit distance since it describes the expansion through the relationship between velocity and distance: $\dot{l} = Hl$, or in more familiar notation $v = Hr$.

We define the *redshift* to a galaxy at distance l to be

$$1 + z = \frac{1}{a} \tag{9}$$

When we look at a distant galaxy we are looking at it as it was in the past (because of the finite light travel time). At the time we are seeing it, the scale factor $a(t)$ was smaller than the present value ($a_0 = 1$). It can easily be shown that the recession velocity we measure from the shift in the spectral lines is just cz, in other words, the quantities z appearing in equations (3) and (9) are the same thing.

1.1.3 Important quantities: H_0, Ω_0, ρ_c

At this point it is convenient to introduce some fundamental definitions. Hubble's expansion law states that the recession velocity of a galaxy is proportional to its distance from the observer, in other words $\dot{l} \propto l_0$. The constant of proportionality (the cosmic expansion rate) is the present value of the Hubble parameter:

$$H_0 = \frac{\dot{l}_0}{l_0} = \frac{\dot{a}_0}{a_0}, \tag{10}$$

H_0, the present value of the Hubble Parameter, is usually called "Hubble's Constant".

There is an important value of the density, ρ_c, that can be derived from the Hubble parameter (the Hubble parameter has dimensions $[\text{time}]^{-1}$). This is the density such that a uniform self-gravitating sphere of density ρ_c isotropically expanding at rate H has equal kinetic and gravitational potential energies:

$$\rho_c = \frac{3H^2}{8\pi G}. \tag{11}$$

Since H is a function of time, then so is ρ_c.

We can measure the density of the Universe in terms of ρ_c by introducing the *density parameter* Ω:

$$\Omega = \frac{\rho}{\rho_c}. \tag{12}$$

Note that Ω also depends on time and we shall denote the present day value of Ω by Ω_0. There may be a mixture of different type of matter in the universe that make up the total density ρ. We may think, for example, of baryons, photons and perhaps some exotic elementary particles. Each of these individually has a density that can be normalized relative to ρ_c, thus each species has its own Ω. We will, for example, denote the contribution of Baryonic material to the total cosmic density by Ω_B.

The density ρ_c has a special significance. A universe whose density is ρ_c when its expansion rate is H is referred to as an *Einstein de Sitter* universe. This model clearly has $\Omega = 1$ at all times. The expansion rate of such a universe is fixed by the density. Model universes that are denser than $\rho_c = 3H^2/8\pi G$ when their expansion rate is H will stop expanding and contract down to future singularity. Models that are less dense will expand forever. The $\Omega = 1$ universe is a limiting case dividing two classes of behaviour and that is why the parametrization of the density in terms of ρ_c is so useful. The behaviour of the various model universes as a function of Ω can be seen by looking at dynamical equation for the expansion factor $a(t)$.

Equations (5) and (7) for $a(t)$ can be shown to integrate to

$$\left(\frac{\dot{a}}{a}\right)^2 = \Omega_0 H_0^2 a^{-3} - H_0(\Omega_0 - 1)a^{-2}. \tag{13}$$

The integration constants have been derived using the boundary condition that $a(t) \to 0$ as $t \to 0, (\dot{a}/a)_0 = H_0$ and that the present density of matter is $\rho_0 = \Omega_0 \rho_c$. The standard textbooks referred to above give the solutions of this equation for general values of Ω_0. It is sufficient here to note that the case $\Omega_0 = 1$ simplifies the right hand side of this equation and the solution is then particularly simple

$$a(t) \propto t^{\frac{2}{3}} \qquad \Omega_0 = 1.$$

Since $a(t) = (1+z)^{-1}$ this tells us that when we look back to a redshift z in Einstein de Sitter universe we are seeing the universe when its age is a fraction $t/t_0 = (1+z)^{-\frac{3}{2}}$ of its present age, t_0.

1.2 The Hubble Parameter h

Determining the Hubble constant, H_0, requires that we have a way of getting the distance to galaxies independently of their redshifts. The history of determining the extragalactic distance scale is in itself a fascinating subject (Rowan-Robinson, 1986) and even today there is considerable uncertainty. There seems to be two distinct bodies of opinion, one clustering its estimates of H_0 around 50 km s^{-1} Mpc^{-1} and the other around 80 km s^{-1} Mpc^{-1}. We shall absorb this ignorance into a "Hubble parameter" h defined so that

$$H_0 = 100h \text{ km s}^{-1} \text{ Mpc}^{-1}.$$

So all distances quoted will contain the quantity h, and the reader is invited to substitute her/his favourite value.

It is probably safer in practise to use radial velocity to express distances. This reflects the Hubble law and so when we say a galaxy is at a distance of $30h^{-1}$ Mpc. we could equally well say it is at a distance of 3000 km s^{-1}. This is fine, but it may look a bit strange to say that a void has a diameter of 5000 km s^{-1}, or to say that the galaxy clustering correlation function drops to unity on a scale of 500 km s^{-1}.

The present value of the Hubble Constant, H_0, and the density parameter, Ω_0, together determine the present age of the universe. In the case of an $\Omega_0 < 1$ universe:

$$
\begin{aligned}
t_0 &= \frac{1}{H_0}\left[\frac{1}{(1-\Omega_0)} - \frac{2}{\Omega_0}\frac{1}{(1-\Omega_0)^{3/2}}\cosh^{-1}\left(\frac{2}{\Omega_0}-1\right)\right], & \Omega_0 < 1, \\
&\to H_0^{-1}, & \Omega_0 \to 0, \\
&\to \frac{2}{3}H_0^{-1}, & \Omega_0 \to 1,
\end{aligned}
$$

It is certain that there should not be any objects older than this in the Universe, so determining ages is an important way of constraining the values of H_0 and Ω_0. It seems that the oldest known stellar systems for which we can determine ages have ages in excess of 16 Gyr. (Sandage and Cacciari, 1990). If we accept this value, then we see that an $\Omega_0 = 1$ universe is always too young unless H_0 is considerably lower than any of the values so far put forward. An open universe with $\Omega_0 < 0.1$ can work provided H_0 is at the lower end of the suggested range of values.

What are we to make of this? That neither age determinations of star clusters nor the extragalactic distance scale can be relied on, with the latter probably being the most uncertain. Introducing a cosmological constant would of course help.

1.3 The Cosmological Constant Λ

The cosmological constant has been recently reviewed by Peebles (1988) and by Weinberg (1989). (See also Klapdor and Grotz, 1986). The wealth cosmological models that can arise through simply introducing Λ is discussed in the great ancient book by Tolman (1934).

1.3.1 Expansion with Λ

Einstein presented a version of his famous field equations containing an additional constant of nature, the Cosmological Constant, Λ. The consequence of introducing this *ad hoc* term into the equations can be seen by studying the dynamical equations with the Λ-term. In the simplest case of zero pressure (which approximates the present day circumstances):

$$
\frac{1}{a}\frac{d^2a}{dt^2} = -\frac{4\pi G}{3}\rho + \frac{\Lambda}{3}.
$$

This is the generalization of (5, 6, 7).

A positive Λ-term increases the acceleration of the expansion, and gives rise to the possibility that the two terms on the right hand side of (14) can at some time during the evolution balance:

$$4\pi G\rho_\Lambda = \Lambda, \qquad p = 0.$$

This "zero-acceleration" time can easily be shown to occur at a redshift z_Λ given by

$$1 + z_\Lambda = \left(\frac{2\Lambda}{3\Omega_0 H_0^2}\right)^{\frac{1}{3}}$$

At a time before the discovery of the cosmic expansion by Hubble, Einstein proposed that the Universe could be static if there was such a Λ term, and that the value of Λ would determine the cosmic density. The later discovery of the cosmic expansion did not, however, cause people to drop the Λ term from the equations.

We can integrate (14) once. Using the conservation of matter expressed as $\rho = \rho a^{-3}$ (equation 7), we get

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G\rho(t)}{3} - \frac{k}{a^2} + \frac{\Lambda}{3}. \qquad (14)$$

The Λ version of equation (13) follows by evaluating the integration constant k (the *curvature constant*) at the present epoch:

$$k = H_0^2(\Omega_0 - 1) + \frac{\lambda}{3}, \qquad (15)$$

to give

$$\left(\frac{\dot{a}}{a}\right)^2 = \Omega_0 H_0^2 a^{-3} - \left[\frac{\Lambda}{3} + H_0^2(\Omega_0 - 1)\right] a^{-2} + \frac{\Lambda}{3}. \qquad (16)$$

A universe with $k = 0$ is said to be "flat". If $\Lambda = 0$, then the flat universe is a $\Omega = 1$ universe (the Einstein de Sitter model). There is a considerable body of opinion in favour of $k = 0$, but until the idea of an early "inflationary" phase of expansion was introduced by Guth, the reasons for favouring such a model were largely aesthetic. Inflation generally demands $k = 0$. (There was an argument that large scale structures would have to form very early ($z > \Omega_0^{-1}$) if Ω_0 were small. Since we see the quasar population growing to a maximum more recently than a redshift of 3, this would suggest a relatively recent formation epoch for galaxy clusters if we could think of some argument relating QSO activity to the origin of clusters! We have in fact no direct evidence as to when the first large scale structures formed.)

1.3.2 The λ parameter

Note that we can introduce a dimensionless measure of Λ:

$$\lambda = \frac{\Lambda}{3H_0^2}. \tag{17}$$

Then the curvature constant, k becomes

$$k = H_0^2(\lambda + \Omega_0 - 1).$$

The $k = 0$ flat universe such as implied by inflationary theories therefore has

$$\lambda_0 + \Omega_0 = 1.$$

If we argued that the dark matter was all baryonic, contributing $\Omega_0 = 0.2$, then we would need $\lambda = 0.8$ for consistency with standard inflationary scenarios.

The coasting redshift in terms of λ is

$$1 + z_\Lambda = \left(\frac{2\lambda}{\Omega_0}\right)^{\frac{1}{3}}. \tag{18}$$

For values of λ such as those described above for a flat universe we see a coasting period at relatively recent redshifts $z_\Lambda \simeq 1 - 2$.

1.3.3 Why introduce Λ?

Current thinking on the issue of whether Λ should be there or not varies over a short timescale of a few years. There is certainly no observational evidence for including the Λ term in the equations. From the point of view of our limited understanding of the status of the Einstein Field Equations in Quantum Field theory, there is every reason to want it to be exactly zero. However, it brings an extra parameter into the cosmological model and an extra degree of freedom with which to fit the observations. Thus we often see it being brought in at a time when there appear to be difficulties in explaining a set of observations.

A notable example of "wheeling in the cosmological constant" was in the case of explaining why there were so many more Quasars having a redshift near to 2 than might have been expected. This could be explained by putting the coasting period at a redshift of 2 (Petrosian, Salpeter and Szekeres, 1967). More recently, it has been noticed that certain standard models for galaxy formation do not have enough clustering on the largest scales. This might be explained in part by the dynamical influence Λ could have on the formation of large scale structure (Efstathiou, Sutherland and Maddox, 1990; Lahav et al. 1991).

1.4 The Value of Ω_0

The luminous matter itself accounts for only $\Omega_{\rm lum} \sim 0.005h^2$ (Faber and Gallagher, 1979). This was discovered long ago to be insufficient to account for either the flat rotation curves of disk galaxies (the dark massive halo problem, Rubin (1988)), or for the velocity dispersions of groups and clusters of galaxies (the virial mass discrepancy problem, Zwicky (1933)). It later became apparent from cosmic nucleosynthesis arguments that the baryonic density of the universe was substantially higher than the density inferred from the luminous material. There is "dark (nonluminous) baryonic material" in some form or other, perhaps warm gas, or even very low luminosity stars. The amount of baryonic dark matter inferred from nucleosynthesis appears to be just about enough to explain the cluster virial mass discrepancy problem in most clusters of galaxies. However, this would not be sufficient to make $\Omega_0 = 1$. Whether we care about $\Omega_0 \neq 1$ is a central issue of cosmology, so I shall discuss briefly the various ways we get at Ω_0 and why we should strive to get $\Omega_0 = 1$.

But first a word of caution which we will continually return to throughout these lectures. Determinations of Ω_0 are frustrated by the fact that Ω_0 describes the quantity of gravitating matter in the universe, whereas we only see the luminous material which is but a fraction of the total mass density. If the luminosity density were everywhere proportional to the mass density, this would not prove a problem since it would only be necessary to discover what the sealing factor is. However, it is evident that the mass and light are distributed differently on different scales and some other hypothesis is needed.

The simplest hypothesis of this kind is that the *fluctuations* in mass density about the mean are proportional to the *fluctuations* in light density. The constant of proportionality is referred to as the *biasing* *parameter* and it is denoted by the symbol b. We shall encounter this frequently in what follows. Note that the constancy of the biasing parameter is merely a simplifying hypothesis, the actual situation could be far more complicated.

1.4.1 The Deceleration Parameter q_0

The central task of classical cosmology was to determine the cosmic expansion rate, H_0 and the *deceleration parameter* q_0:

$$H_0 = \frac{\dot{a}_0}{a_0}, \quad q_0 = -H_0^{-2}\frac{1}{a_0}\left(\frac{d^2a}{dt^2}\right)_0. \tag{19}$$

H_0 was seen as the slope of the velocity-distance relationship and q_0 as the deviation from the linear Hubble law, its curvature, due to the gravitational deceleration of the cosmic expansion.

Note that by virtue of equation (5) and the definition of Ω_0 (equations (11) and (12))

$$q_0 = \frac{1}{2}\Omega_0 + \lambda, \tag{20}$$

with $\lambda = \Lambda/3H_0^2$. This relationship between Ω_0 and q_0 holds only as long as equations (5, 6, 7) or (14) are valid; that is, provided there is no cosmic pressure. The Einstein de Sitter universe has $q_0 = 1/2$ (since $\Lambda = 0$ and $\Omega_0 = 1$).

We can calculate the relationships between the redshift of a galaxy and various observed properties such as brightness, look-back time and surface brightness. For example, the *look-back time*, measuring the time elapsed since photon was emitted at time $t_{\rm E}$, to redshift z is

$$t_0 - t_{\rm E} = \frac{z}{H_0}\left[1 - (1 + \frac{1}{2}q_0)z + \ldots\right], \tag{21}$$

for small z.

The apparent brightness l is related to the intrinsic luminosity L by

$$l = \frac{L}{4\pi}\left(\frac{H_0}{c}\right)^2\left[\frac{q_0^2}{q_0 z + (q_0 - 1)(1 + 2q_0 z)^{\frac{1}{2}}}\right]^2, \tag{22}$$

and for not too distant galaxies ($z < 1$), this simplifies to

$$l = \frac{L}{4\pi}\left(\frac{H_0}{cz}\right)^2 [1 + (q_0 - 1)z + \ldots]. \tag{23}$$

This expression is also exact in the limits $q_0 = 0$ and $q_0 = 1$. The first terms are simply the standard r^{-2} inverse square law, the correction due to the q_0 term is due to the deceleration of the expansion. In astronomical units this is

$$m = M + 25 - 5\log_{10} H_0 + 5\log_{10} cz + 1.086(1 - q_0)z + \ldots, \tag{24}$$

where m is the apparent magnitude of a galaxy of absolute magnitude M seen at a redshift z. (Technically, these are the luminosities or magnitudes integrated over the whole spectrum of the emitted light. If the measurements are done in a restricted spectral band, then other terms come into this relationship, these are the so-called K-correction terms). This expresses the Hubble Law directly in terms of a magnitude-redshift relationship.

Note that any intrinsic evolution of the quantity L (or the absolute magnitude M) will introduce non-geometric effects into the relationship and so confuse the determination of q_0. We can approximate this by assuming that the luminosity evolves as

$$L(t) = L_0[1 + \alpha(t - t_0)], \tag{25}$$

when expressed as a function of look-back time $t - t_0$. Relating look-back time to redshift then yields

$$l = \frac{L}{4\pi}\left(\frac{H_0}{cz}\right)^2 [1 + (q_0 - 1)z - \alpha H_0^{-1} z + \ldots]. \tag{26}$$

showing an extra linear dependence on z. Thus if this relationship is used to measure q_0, the (unknown) evolutionary correction biases q_0 downward by αH_0^{-1}.

In the small z limit, we can also calculate the number of galaxies $N(m)$ we would see in a galaxy survey down to apparent magnitude m, again under the assumption that there are no evolutionary effects. This is the classical *number-magnitude relationship.*

In the pre-1965 days of cosmology the central issue in cosmology was the values of H_0 and q_0. Cosmology was simply "a search for two numbers". Today, that view has changed. Cosmology is properly a branch of physics and the values of these two parameters are regarded simply as important parameters that observations will eventually determine to characterize our Universe.

Unfortunately, the program of measuring the curvature of the Hubble Law directly has not provided any strong constraints on q_0. This is largely because the curvature of the relationship is influenced by non geometric effects (galaxy luminosities evolve with time in an unknown way) and because there is considerable scatter in the magnitude-redshift diagram. Indeed, the tendency today is to use the Hubble diagram and the number-magnitude relationship together to determine the evolutionary history of galaxies! (See Guideroni and Rocca-Volmerange, 1990; Rocca-Volmerange and Guideroni, 1990). As we shall see below, there is a possibility that this will yield limits on Ω_0 and Λ as a by-product since faint galaxy counts are sensitive to Ω_0.

1.4.2 The classical approach again

In order to discriminate cosmological models, the magnitude redshift relationship needs a large sample of redshifts out to at least $z = 0.5$ and preferably as far beyond $z = 1$ as possible. Loh and Spillar (1986) have used a galaxy survey to determine approximate redshifts for $\sim$ 1000 galaxies out to a redshift of $z \sim 1$. On the basis of this survey, they look at the redshift-volume relationship and conclude that $\Omega_0 = 0.9 \pm 0.3$ if $\Lambda = 0$. Caditz and Petrosian (1989) argue that the luminosity function history assumed by Loh and Spillar is not consistent with their data. Taking this into account, Caditz and Petrosian derive $\Omega_0 \approx 0.2$ with considerable uncertainty due to such things as incompleteness of the sample. Yoshii and Takahara (1989) make a detailed model for the luminosity evolution based on merger driven evolution and discuss the problems associated with such methods of getting at Ω_0.

The number magnitude relationship provides an alternative probe of cosmological models and galaxy evolution and has generated a great deal of interest since we can now survey galaxies down to extremely faint magnitudes in many wavebands. In recent years we have seen faint galaxy counts by Tyson (1988) and by Jones et al. (1991) and Metcalf et al. (1991). The latter surveys penetrate to B-magnitudes $B < 25$. The interpretation of such counts and the galaxy evolution models that are used have been discussed by Koo (1990) and by Guideroni and Rocca-Volmerange (1990). It seems that the present data in the R and B bands can be largely understood in terms of current models of galactic evolution. However, Cowie (1991) has recently presented some infrared counts of galaxies which confuse the situation somewhat by appearing to demand a non-zero cosmological constant! This is also the conclusion of the analysis of counts by Fugikita et al. (1990).

1.4.3 Cosmic Nucleosynthesis

Cosmic nucleosynthesis sets strong bounds on the amount of baryonic material in the Universe (Boesgard and Steigman, 1985; Pagel 1991a,b). Standard Big Bang nucleosynthesis implies that

$$0.011 < \Omega_B h^2 < 0.026, \tag{27}$$

where Ω_B is the contribution of baryons to the total mass density. (See chapter 4 of the Kolb and Turner (1990) book for an excellent discussion of this). There is a need already here to have ten times as much mass in the baryonic dark matter as is accounted by the luminous mass in galaxies.

The nucleosynthesis question is fully discussed elsewhere in this volume. There is a couple point that should be emphasised here. The low baryonic density implied by nucleosynthesis causes a problem in the hydrogen-helium cooling of the pregalactic gas: the density may simply be too low. This is a point about star formation, but it does have a bearing on what we observe on the largest scales since we can only observe what is luminous.

1.4.4 Ω_0 from Hubble flow deviations

The large scale peculiar motions of galaxies are clearly related to the density inhomogeneities, since it is those inhomogeneities that give rise to the peculiar motions. Ω is involved in that relationship and so in principle it could be estimated by comparing density excursions with peculiar velocities. The problem arises because we cannot directly observe the fluctuations in mass density, but only the fluctuations in luminosity density. Another parameter relating mass density fluctuations to luminosity density fluctuations comes into the game. This parameter, b, is called the *bias parameter* and it might depend on the location, the morphological type of the galaxies involved, or any number of other things. In this spirit of ignorance the simplest assumption we can make is that b is a universal constant. Then we can in principle determine the combination $\Omega_0 b^{-5/3}$. We shall have more to say about this below (see sections 2.1.3 and 4.2.1).

On the assumption that light traces mass ($b = 1$), most dynamical determinations of Ω_0 converge on $0.1 < \Omega_0 < 0.3$ (Peebles, 1987; Shanks et al., 1989). Stavely-Smith and Davies (1989) report $\Omega_0 = 0.08 \pm 0.05$. (The latter authors remark that some 'biasing' is demanded by their data, bringing Ω_0 up to at least 0.25.)

More recently, deep redshift surveys of galaxy samples drawn from the IRAS catalog have provided another route to Ω_0. The limits from these surveys involve another parameter, b, the biasing parameter, whose value is largely unknown (and may not even be a constant):

$$\frac{\Omega_0}{b^{5/3}} \approx 1.0,$$

with large error bars. More will be said about this approach later on (section 2.3.2).

1.5 $\Omega = 1$, Dark Matter and Inflation

There is no compelling direct observational evidence for $\Omega_0 = 1$. The driving force behind the notion that $\Omega_0 = 1$ is undoubtedly the inflationary picture for the early universe (see the review of Brandenburger, 1990). Not only does this picture have appeal in providing answers to some fundamental questions (like the horizon problem), but it seems almost inevitable from the point of view of our present knowledge of high energy physics. That weighs more strongly in favour of adopting $\Omega_0 = 1$ than the lack of any obvious candidate particle weighs against the notion.

1.5.1 Flatness and Inflation

It is interesting to write down and solve the equation for the evolution of the density parameter $\Omega(t)$ with time (Ducloux, 1989). Suppose the matter in the universe has density ρ and pressure p such that

$$\beta = \frac{\rho + \frac{3p}{c^2}}{2\rho} = \begin{cases} -1, & \text{if } p = -\rho c^2; \\ \frac{1}{2}, & \text{if } p = 0; \\ 1, & \text{if } p = \frac{1}{3}\rho c^2. \end{cases}$$

The case $p = 0, \beta = 1/2$ is relevant to the current epoch, while the radiation gas case $p = 1/3\rho c^2, \beta = 1$ is relevant to the early universe.

The interesting very early universe case $p = -\rho c^2$ corresponds to $\beta = -1$.

A not inconsiderable amount of work gives the evolution of Ω as

$$\dot{\Omega} \propto 2\beta\Omega^{3/2\beta}(\Omega - 1)^{\beta - 1}.$$

The simplest case $\beta = 1$ is relevant to the current era and the solution is then trivial:

$$\Omega = \frac{1}{1 + \frac{t}{t_0}(\Omega_0^{-\frac{1}{2}} - 1)^2}$$

Thus in order to get $\Omega_0 = 0.1$ today, we need $\Omega_P = 1 - 10^{-60}$ at the Planck time $t_P \sim 10^{-60} t_0$. It is the fact that Ω should have been so incredibly close to 1.0 initially that is referred to as the "fine tuning" problem. The argument goes then that it was so close it must (sic) have been exactly 1.000 ... . The alternative is to seek a mechanism whereby such a value might be generated. The mechanism is "inflation".

In its simplest form the idea of inflation is as follows. If at an early stage the universe had equation of state

$$p_V = -\rho_V c^2,$$

the cosmic expansion would be exponential:

$$\begin{aligned} a &\propto e^{H_V t} \\ H_V &= \frac{8\pi}{3} G \rho_V \end{aligned}$$

The fact that the pressure p_V is negative is a consequence of the physics of the vacuum at the high temperatures prevailing in the early universe.

The exponential expansion phase is referred to as the "de Sitter phase" and would continue for as long as the material had this peculiar equation of state. The universe then makes a transition to an expansion for a "normal" equation of state. It turns out that during this "de Sitter" expansion phase, very distant parts of the universe are causally connected. This is presumed to be an "explanation" for the flatness problem, the idea being that all anisotropies and inhomogeneities disappear during this phase of phenomenal expansion.

The other side of the coin is that it is necessary to generate some primordial inhomogeneities that will eventually give rise to the formation of galaxies and large scale structure. These must be generated during or after the inflationary era and are generally thought to arise out of quantum fluctuations in the vacuum state. In the simplest models the spectrum of fluctuations is the Harrison-Zel'dovich spectrum with Gaussian distributed fluctuations (Guth and Pi, 1985). We now appear to have evidence that this spectrum does not have enough large scale power to explain the observations of large scale structure. (See Kashlinsky and Jones (1991) for arguments that the spectrum is not Harrison-Zel'dovich).

There is however no lack of alternate (albeit somewhat *ad hoc*) model which allow us to get around the problem of the lack of large scale power in the Harrison-Zel'dovich spectrum. One of the most plausible ways around this is by generating non-Gaussian fluctuations. Chaotic inflation (Linde, 1984 Linde and Mukhanov, 1987) can generate non-Gaussian fluctuation (Matarrese, Ortolan and Lucchin, 1989; Yi, Vishniac and Mineshige, 1991). Strong claims have been made for models involving "global texture" as these also generate non-Gaussian initial fluctuations (Spergel et al., 1991).

2 INHOMOGENEOUS UNIVERSE — OBSERVATIONS

The inhomogeneity of the Universe has been a major aspect of cosmology over the last 25 years. We have learned a great deal, especially from redshift surveys, and although things turn out to be fairly complicated in the sense that the Universe is not simple a pile of clusters distributed at random, nevertheless possesses some systematics upon which we can build models. The structures we have seen on the largest scales seem to be traced equally by galaxy samples drawn from quite different catalogues: optical catalogues (de Lapparent et al., 1986, 1988), infrared catalogues (Babul and Postman, 1990) or even catalogues of dwarf galaxies (Thuan et al, 1991).

In this section, I review the observed character of the clustering and the various attempts that have been made to quantify those visual impressions. Inevitably, much of the interpretation is predicated on notions developed through theories so it is almost impossible to discuss the observational data without reference to the theory! The theory comes in the following section. However, I shall try to provide interpretations of the data that are largely model independent.

2.1 Preliminaries

Before discussing the data, we need a few basic notions such as correlation functions, bias parameters and the like. This subsection introduces these at a simple level, they are discussed in more detail later on.

2.1.1 2-Point Correlation Functions

The two-point clustering correlation function has been the mainstay of clustering studies for over fifteen years. Its importance in cosmology has been fully discussed by Peebles (*LSSU*: 1980).

The 2-point correlation function as used in astrophysics describes one way in which the actual distribution of galaxies deviates from a simple Poisson distribution. There are other descriptors like three point correlation functions, the topological genus and so on; we shall come to those in more detail below (Section 3).

There are two sorts of 2-point function. One describing the clustering as projected on the sky, thus describing the angular distribution of galaxies in a typical galaxy catalogue. This is called the *angular 2-point correlation function* and is generally denoted by $w(\theta)$. The other describes the clustering in space and is called *the spatial 2-point correlation function*. We frequently omit the word "spatial". The (spatial) 2-point correlation function is generally denoted by $\xi(r)$.

In order to provide a mathematical definition of the correlation function we will only consider the spatial 2-point function, the definition of the angular function follows similarly.

Consider a given galaxy in a homogeneous Poisson-distributed sample of galaxies, then the probability of finding another galaxy in a small element of volume δV at a distance r would be $\delta P = n\delta V$, where n is the mean number density of galaxies. If the sample is clustered then the probability will be different and will be expressible as

$$\delta P = n[1 + \xi(r)]\delta V, \tag{28}$$

for some function $\xi(r)$ satisfying the conditions

$$\xi(\mathbf{r}) \quad \geq \quad -1,$$

$$\xi(\mathbf{r}) \quad \rightarrow \quad 0, \quad |\mathbf{r}| \rightarrow \mathbf{0} \tag{29}$$

The first condition is essential since probabilities are positive, and the second is required in order that a mean density exist for the sample.

It is customary to make the assumption that the two point function is isotropic: it depends only on the distance between two points and not the direction of the line joining them:

$$\xi(\mathbf{r}) = \xi(r).$$

This is a reasonable but untested hypothesis.

In practice, the correlation function is estimated simply by counting the number of pairs within volumes around galaxies in the sample, and comparing that with the number that would be expected on the basis of a Poisson distributed sample having the same total population. There are subtleties however due to the fact that galaxies lying near the boundary of the sample volume have their neighbours censored by the bounding volume.

One method discussed by Rivolo (1986) is to use the estimator

$$1 + \xi(r) = \frac{1}{N} \sum_{i=1}^{N} \frac{N_i(r)}{nV_i(r)}, \tag{30}$$

where N is the total number of galaxies in the sample and n is their number density. $N_i(r)$ is the number of galaxies lying in a shell of thickness δr from the ith galaxy, and $V_i(r)$ is the volume of the shell lying within the sample volume. (So $N_i(r)$ is being compared with $nV_i(r)$, the Poisson-expected number lying in the shell). Note that n is usually taken to be the sample mean, but if there is an alternative (and better) way of estimating the mean density, the alternative should be used.

An alternative strategy to calculating $\xi(r)$ for a catalogue of N_G galaxies is to put down N_R points at random in the survey volume and compare the number of pairs of galaxies $n_{GG}(r)$ having separation r with the number of pairs $n_{RG}(r)$ consisting of a random point and a galaxy, separated by the same

$$1 + \xi(r) = \frac{n_{GG}(r)}{n_{GR}(r)} \frac{N_R}{N_G}, \tag{31}$$

(Davies et al. 1988).

(As an aside it is worth commenting that some authors (eg. Pietronero, 1987) have advocated using a "structure function" for a distribution of

N galaxies situated at points $\mathbf{r_i} : \Gamma(r) = N^1 \sum n(\mathbf{r} + \mathbf{r_i})$. This is analogous to the correlation function, but has the claimed advantage that it does not depend explicitly in its *calculation* on the mean density for the sample. The value of the function is however strongly density dependent, as can be seen for a Poisson distribution where $\xi(r) = 0$ and $\Gamma(r) = \rho$. The idea is that one would be able to study the clustering without the assumption that the universe is homogeneous. Given that there is considerable evidence that the universe is indeed homogeneous in the large, this may seem somewhat unnecessary.)

The two point correlation function for the distribution of galaxies has a roughly power law behaviour on scales $R < 10h^{-1}$ Mpc., with a slope of -1.77:

$$\xi(r) = \left(\frac{r}{r_0}\right)^{-1.77}, \qquad r < 10h^{-1} \text{ Mpc.}, \tag{32}$$

$$r_0 \sim 5h^{-1} \text{ Mpc.}$$

This is frequently referred to as "the 1.8 power law". What happens beyond $10h^{-1}$ Mpc, is somewhat contentious. It certainly falls below the power law behaviour, but it is not even clear whether it falls to negative values at any scale where it is measurable. What is notable is that the two-point correlation function is of negligible amplitude on those scales ($R \sim 20h^{-1}$ Mpc.) where the structure revealed in the redshift surveys (de Lapparent, Geller and Huchra, 1986) is most dramatic. (If there were enough galaxies that we could determine the values of $\xi(r)$ on these scales, its precise shape would indeed contain information about the large scale clustering. We are however constrained by sample discreteness.)

It should be remarked that the low amplitude of the two-point function on these large scales is consistent with the fact that the universe, if smoothed over such scales, would show little structure. We see the structure by virtue of what is happening on the small scales and in particular how the small scale structures relate to one another. The inadequacy of the 2-point function in describing what is seen on the largest scales has motivated people to look at other ways of describing the large scale structure.

The accuracy with which the two-point correlation function is determined in redshift surveys has been questioned: Einasto, Klypin and Saar (1986) argued that r_0 depended systematically on the depth of the sample, though this is probably a consequence of a bias introduced by luminosity segregation (Martínez and Jones, 1990).

2.1.2 The Power Spectrum

Formally, the Power Spectrum of a distribution of points is defined as the Fourier transform of the two-point correlation function:

$$\begin{aligned} \mathcal{P}(\mathbf{k}) &= \frac{n}{(2\pi)^{3/2}} \int \xi(s) e^{\mathbf{ik}\cdot\mathbf{s}} d^3\mathbf{s} \\ &= \left(\frac{2}{\pi}\right)^{1/2} n \int_0^\infty \xi(s) \left(\frac{\sin ks}{ks}\right) s^2 ds, \end{aligned} \tag{33}$$

where the last equality follows because $\xi(\mathbf{r}) = \xi(\mathbf{r})$ is direction independent. ($k = |\mathbf{k}|$). These relationships are formally invertible, so given $\mathcal{P}(k)$ it is possible to get $\xi(r)$.

So why introduce the power spectrum? The reason can be seen from the following argument. Suppose that the density of galaxies in a volume V is $n(\mathbf{r})$, and that the mean of this is n (so the total number of galaxies is nV).

Then the fluctuating density field $n(\mathbf{r}) - n$ can be decomposed into Fourier plane waves:

$$n(\mathbf{r}) - n = \sum_{k \neq 0} n_{\mathbf{k}} e^{\mathbf{ik}\cdot\mathbf{r}}, \tag{34}$$

where the sum extends over all wavenumbers than fit in the volume. It can then be shown that

$$\mathcal{P}(\mathbf{k}) = \frac{nV}{(2\pi)^{3/2}} \frac{|n_{\mathbf{k}}|^2}{n^2}, \tag{35}$$

and that (by Fourier transform relationship between $\mathcal{P}(k)$ and $\xi(r)$):

$$\xi(r) = \sum_{k \neq 0} \frac{|n_{\mathbf{k}}|^2}{n^2} e^{-\mathbf{ik}\cdot\mathbf{r}}. \tag{36}$$

The distribution of the amplitudes of each Fourier component of the fluctuating density field is determined by the physical processes that generated the fluctuations in the first place. As long as the amplitudes are small and linear theory applies, the evolution of these Fourier components is independent and determined by the physical processes in the early universe. So decomposing the density field into a set of Fourier components is useful. What the power spectrum then tells us is the distribution of the mean square amplitudes of these components. The power spectrum is the contribution of each Fourier mode to the total variance of the density fluctuations.

The appearance of the $|n_{\mathbf{k}}|^2$ term is interesting. It tells us that $\mathcal{P}(k)$ only contains information about the amplitudes of the Fourier components of the fluctuating density field. $\mathcal{P}(k)$ contains no phase information. Consequently, the 2-point correlation function does not contain this information either. Many quite different distributions can have the same power spectrum and correlation function. Consider the example of a density field that is uniform on the faces of a cubic lattice, and zero elsewhere. That can be written as a Fourier series and the Fourier component have highly correlated phases. Randomize the phases and the density distribution looks inhomogeneous and quite disordered, yet the correlation function and power spectrum remain unchanged.

2.1.3 Biasing

As a final technical point we should look at the concept of *biasing* that comes in when one wishes to compare the distribution of matter (some of which may be dark) with the distribution of luminous galaxies. Light does not necessarily trace mass and so the clustering properties of the light distribution may be quite different from the clustering properties of the mass distribution.

The need to relate the mass and light distributions arises in two situations. In the first place one may wish to make a comparison between the predictions of an N-body model for galaxy clustering with the observed distribution. At present the N-body models simply describe the distribution of the gravitating matter and some hypothesis is needed to say which material particles are luminous. Ideally, really sophisticated N-body models would incorporate details of the star formation process and make such a hypothesis unnecessary. In the second place, we may simply wish to infer the mass distribution from the observed light distribution in order to relate the observed velocity fields to the matter distribution that generated them.

It would be bizarre indeed if the distributions of mass and light were not related and the simplest hypothesis is that the fluctuations in the mass distribution are proportional to the fluctuations in the luminosity distribution:

$$\frac{\delta n}{n} = b\frac{\delta\rho}{\rho}. \tag{37}$$

Here n represents the mean density of galaxies (the luminous material) and δn the fluctuations in the galaxy density (which will be position dependent). ρ represents the mean mass density and $\delta\rho$ the fluctuations in mass density.

This assumption is probably a considerable simplification of the real physics, but we have to start somewhere. In fact, b is generally chosen to be a constant, independent of position or scale, but possibly dependent on the morphological type of galaxy that makes up the luminous sample. This is forced on us because whereas all galaxies seem to trace out the same large scale distribution, some are more clustered than others. For example, the elliptical galaxies seem to be more highly clustered than spirals (they occupy the denser regions of the universe). So the bias parameter for ellipticals must be somewhat larger than that for spirals.

The bias parameter plays a central role in relating the deviations from pure Hubble flow (which are driven by the total matter distribution) with measures of the clustering of galaxies (which depend on where the luminous material happens to be). It can be shown (see Peebles, LSSU equations (8.2) and (14.2)) that in linear theory the relationship between the peculiar velocity field $\mathbf{v}$ and the fluctuating gravitational force $\mathbf{g}$ is

$$\mathbf{v} = \frac{2}{3H\Omega^{0.4}}\,\mathbf{g}, \tag{38}$$

where, by virtue of the perturbed Poisson equation, the fluctuating force $\mathbf{g}$ is related to the distribution of relative density fluctuations $\delta(\mathbf{x}) = \delta\rho/\rho$:

$$\mathbf{g}(\mathbf{x}) = Ga(t)\Omega\rho_c \int \frac{\mathbf{x}' - \mathbf{x}}{|\mathbf{x}' - \mathbf{x}|^3}\,\delta(\mathbf{x}', t)d^3\mathbf{x}', \tag{39}$$

The mass density fluctuations $\delta\rho/\rho$ are supposed to be related to the fluctuations $\delta n/n$ in the observed galaxy density via the bias parameter b defined above. So what we observe is

$$\mathbf{v} = \frac{2}{3H_0}\,G\rho_c\frac{\Omega_0^{0.6}}{b}\int \frac{\mathbf{x}' - \mathbf{x}}{|\mathbf{x}' - \mathbf{x}|^3}\frac{\delta n(\mathbf{x}')}{n}\,d^3\mathbf{x}' \tag{40}$$

Note that there is a normalization factor $\Omega_0^{0.6}/b$ to be fitted when relating the variations in the luminosity with the peculiar velocities. Note also that decreasing b increases $\mathbf{v}$, for a given luminosity distribution.

The upshot of this is that methods for determining Ω_0 that compare velocity fields with density fluctuations in fact only determine $\Omega_0 b^{-5/3}$, and we need to get b from somewhere else (usually a theoretical prejudice based on an N-body model!).

Note that if we compare two different catalogues we can determine $\Omega_0 b^{-5/3}$, for each catalogue. The ratio of these gives us the ratio of the bias parameters for the two catalogues (Babul and Postman, 1990; Lahav et al., 1990). At least we can reassure ourselves that different galaxy catalogues have different bias parameters!

2.2 Projected Surveys

The first quantitative discussions of comic inhomogeneity came through a study, mainly by Peebles and his collaborators (see LSSU), of various catalogues of the positions of galaxies on the sky. The famous 1.8 power law behaviour of the correlation function was discovered from the analysis of such catalogues, and the consistency of the results from different catalogues was interpreted as strong evidence that the clustering was indeed a cosmological effect (and not due to foreground obscuration, for example) and that the clustering was homogeneous as a function of the depth of the catalogue. Whereas the 1.8 power law could be consistently identified, the break away from that law on larger scales (the cutoff) was harder to identify with confidence. In the analysis of the Lick survey Groth and Peebles (1977) found a cutoff that corresponded rather closely with the angular scale of the individual photographic plates and that lead to some skepticism about exactly where the feature was (de Lapparent et al., 1989).

Recently, Maddox et al. (1990a,c) have created a catalogue of some 2 million galaxies based on scans of the UK Schmidt J-survey plates with the SERC Automatic Plate Measuring machine (APM). The catalogue covers some 4300 square degrees and reaches to magnitude $J = 20.5$. It effectively penetrates to a depth of some $600h^{-1}$ Mpc. The two point correlation function for this catalogue (Maddox et al., 1990b) shows results that are consistent with the Lick catalogue results in the 1.8 power law regime, but the cutoff is quite different. The cutoff is not at all as abrupt as the Lick catalogue analysis seems to imply, and there is substantial power on very large scales. In fact there is so much power on these large scales that it is difficult to reconcile the predictions of the standard cold dark matter numerical simulations with the data.

2.3 Redshift Surveys

In the past years redshift surveys have played a very important role in mapping out the structure of the universe. Today, the number of available galaxy redshifts numbers in the tens of thousands, though these are divided among different galaxy catalogues often having rather different selection criteria. Some of these catalogues are based on optically selected galaxies, and then sometimes by limiting magnitude and at other times by limiting angular diameter. Some catalogues select disk galaxies, some select dwarfs and others select ellipticals. There is an important database based on the IRAS satellite data where candidate galaxies are selected according to diverse spectral classification criteria.

There is then the type of survey to consider: is it an all-sky survey, or is it a sector of sky or a small but deep pencil beam? Pencil beams go to great depths, but lack any transverse spatial resolution. The whole sky surveys have good three dimensional discrimination, but cannot go very deep.

2.3.1 Optical Galaxy Samples

Redshift surveys of galaxies involving thousands of galaxies do not suffer from projection effects and so reveal more clearly the true large scale structure of the universe. However, because the map is in redshift space, it suffers from artifacts such as the "finger of God" effect in which clusters of galaxies appear as long fingers pointing radially towards the observer. It is difficult to correct for these effects, so one must be rather careful when interpreting the apparent structures.

The CfA redshift survey (de Lapparent, Geller and Huchra, 1986, 1988) was the first survey to reveal the structures that nowadays dominate much of our thinking about the large scale structure. That survey, and the corresponding Southern Sky Redshift Survey (da Costa et al., 1988), show the familiar filaments surrounding voids: the "bubble-like" texture of the galaxy distribution. These structures in the galaxy distribution appear on scales where the galaxy-galaxy correlation function is negligible. (This is however not surprising. See the comment on this in section 3.1).

The original surveys have now been extended to other slices and they conclude that the structures are essentially sheet-like, and that the scale of the sheets is limited only by the scale of the survey. The most remarkable feature is the so-called "great wall" (Geller and Huchra, 1989) which appears to be a coherent sheet of galaxies extending over an area of at least $60h^{-1} \times 170h^{-1}$ Mpc. Although the appearance of the Geller-Huchra wall is enhanced by the selection function for the sample, it is clear that such features appear in other deep wide angle surveys such as the extension of the SSRS reported by da Costa (1991). It is also apparent that the regular features being detected in the deep pencil beam surveys (Broadhurst et al., 1990; discussed below) are related to these walls. For a discussion of this see Fong, Hale-Sutton and Shanks (1991).

Great Walls do not bound great voids, but seem to surround collections of smaller voids that are themselves bounded by not-so-great walls. It could be that the great walls are simply lesser features (not-so-great walls) picked out and correlated by eye to build a larger structures. In this case the enhancement by the observational selection function would be important in causing us to recognize the local (Geller-Huchra) Great Wall. Looking at N-body models suggests this kind of effect because it is easy for the brain to identify coherent structures on scales where there is no physical mechanism for generating structure.

The Durham deep redshift sample (Metcalf et al., 1989; Hale-sutton et al., 1989) is confined to a set of narrow solid angles in the sky. The survey samples one galaxy in three down to magnitude $J = 16.8$ and contains 264 redshifts. Analysis of the data reveals a redshift space correlation function having a scale length of $7h^{-1}$ Mpc., beyond which there is a steep break in the slope. There is persistent evidence that the correlation function is not a power law, but has a feature ("shoulder") on scales $2 - 5h^{-1}$ Mpc., though this could be an effect due to the fact that the data is seen in redshift space (Shanks et al., 1989). There is also evidence that the correlation function goes negative around $20h^{-1}$ Mpc., never becoming positive again. However, this may simply be due to the fact that the survey areas appear not to contain any notable galaxy clusters. Confirmation of such results is only possible with very large redshift surveys, covering a substantial solid angle of sky.

Klypin, Karachentsev and Lebedev (1990) have made a small survey in a strip of sky $10' \times 63^\circ$ that overlaps with the de Lapparent slice, but goes twice as deep ($m_B < 17.6$). The sample is small, 283 galaxies, but large enough to be able to display the homogeneity of the universe on the largest scales (an issue that had been raised on a number of occasions (for example by Coleman, Pietronero and Sanders, 1988). Again, as in the Durham survey, there is some evidence that the correlation function goes negative on the largest scales.

2.3.2 Surveys based on the IRAS catalogue

Two important redshift surveys have been based on the IRAS catalogue: one is a complete redshift sample of all sources brighter than a given flux (I shall refer to that simply as the "IRAS survey") and the other (called the "QDOT" survey) is a sparse sample of redshifts in which only 1 galaxy in 6 has its redshift taken. (QDOT is an acronym for the participating institutions: Queen Mary and Westfield College, Durham, Oxford and Toronto). The advantage of QDOT is that it goes deeper than the IRAS survey, but because of the sparse sampling it is shot-noise dominated at larger distances.

The IRAS galaxy catalogue does not include early type galaxies, these do not have strong infrared fluxes, and the intrinsically brighter IRAS galaxies tend to be starburst galaxies. This means that despite the fact that the IRAS galaxies trace all known structures, the observed populations do depend on the local density. It also means that the more distant galaxies in the survey tend to be starburst galaxies and so there may be a systematic and environment dependent bias as a function of depth. Having said that, the sky coverage of the IRAS galaxies is only obscured slightly by the galactic plane and this is a considerable advantage, particularly in view of the fact that some of the key structures in which we are interested seem to lie close to the plane. Taking redshifts of objects in the galactic plane is, however, more difficult, so redshift surveys still have to contend with galactic absorption.

The QDOT data is described by Saunders et al. (1990) (though much of analysis of the sample is presented in Rowan-Robinson et al., 1990) and consists of some 2163 galaxies with $60\mu m$ flux brighter than 0.6Jy. It samples the universe to a redshift of $\simeq 0.1$ and provides useful data out to a distance of some $200h^{-1}$ Mpc. A number of papers have appeared analyzing that data in various ways.

The luminosity function of the QDOT sample is given by Saunders et al. (1990) and this determines the selection function $S(r)$ for the sample. $S(r)$ is the expected number of galaxies in the survey out to a distance r on the assumption that the distribution was Poisson. The pictures of Saunders et al. (1991) and the subsequent analysis of Moore (1991) show that the survey contains all known structures. Thus the QDOT survey does indeed map out the universe, despite the fact that it is a sparse sample of a sparse sample. It should be noted, however, that according to Saunders et al. (1990) the luminosity function of the QDOT galaxies evolves with redshift as

$$\phi(L,z) \propto (1+z)^{6.7}. \tag{41}$$

Their analysis argues that this is consistent with a luminosity evolution for the galaxies:

$$L_z = L \exp\left[\frac{2}{3}Q[1-(1+z)^{-3/2}]\right], \quad Q = 3.2 \pm 1.0, \tag{42}$$

but that there is no evidence for any change in the shape of the luminosity function. The intrinsically brighter galaxies in the catalogue are starburst galaxies and these are the ones seen at the greatest distances.

Efstathiou et al. (1990) and Saunders et al. (1991) look at the counts in cells distribution and compare that with the predictions of the CDM models.

Not surprisingly, they find that the variance of the counts cannot be accommodated by standard CDM, but it should be remarked that on those large scales where there appears to be a discrepancy there may be problems arising out of the sparse sampling procedure (shot noise). Rare but rich areas like the Hercules supercluster complex could also bias the cell counts. Rowan-Robinson et al. (1990) have used the convergence of the microwave background dipole to constrain the value of Ω_0 and find

$$\frac{\Omega_0}{b^{5/3}} = 0.7^{+0.3}_{-0.2}, \tag{43}$$

(see also Kaiser et al., 1991). If we believe $\Omega_0 = 1$ this can be read as saying that the bias parameter is $b = 1.23 \pm 0.23$ for this sample of galaxies. Interestingly, Rowan-Robinson et al. are able to account for the peculiar motion of the local group, in both magnitude and direction, entirely in terms of clusters that can be recognised in the QDOT survey. They do not need to postulate any further unseen masses lying behind the galactic plane. The situation is not unlike that found by Plionis and Valdarmini (1991) who account for the dipole in terms of clusters of galaxies drawn from already existing catalogues. I shall comment further on this in section 2.6.2.

The IRAS survey data is reported by Strauss, Davis, Yahil and Huchra (1990) and consists of redshifts of 2649 IRAS galaxies brighter than 1.9Jy. First results from this survey were discussed by Strauss and Davis (1988) and by Yahil (1988) at the Vatican Study Week (Rubin and Coyne, 1988). Work is in progress on a deeper survey ("IRAS2" for want of a better name) going to 1.2Jy (Fisher et al, 1991), the survey contains some 5300 galaxies having redshifts.

Babul and Postman (1990) compare the distribution of an incomplete redshift survey of IRAS galaxies with the CfA slice (de Lapparent, Geller and Huchra, 1986, 1988). Correlation analysis suggests that the bias parameter for the IRAS sample is a factor 1.6 down on that for the CfA sample: $b_{CfA}/b_{IRAS} \approx 1.6$. Nevertheless the IRAS galaxies do not appear to favour the voids any more than the CfA survey galaxies. This would be easily explained if the IRAS galaxies were merely a subset of the CfA galaxies.

Lahav, Nemiroff and Piran (1990) estimated the ratios of the bias parameters for the IRAS catalog of galaxies and an optical catalog. The two catalogs show different correlation lengths, which reflects the lack of ellipticals in the IRAS catalog and is presumably due to a different level of bias in the catalogs. The two catalogs also provide different estimates for the density parameter Ω_0.

2.3.3 Pencil Beam Surveys

Broadhurst et al. (1990) have combined four deep narrow-angle surveys of galaxy redshifts that give a picture of the distribution to $2000h^{-1}$ Mpc. The interesting outcome of this is an apparent regularity in the distribution of radial velocities on a scale $\sim 120h^{-1}$ Mpc. The authors conclude that 'it is difficult to understand how so many features could maintain organized regularity over such a long baseline'.

The result, if confirmed, is indeed surprising and points to a hitherto unsuspected order in the universe. It is not clear to what extent various numerical models of the formation of large scale structure could explain this. It has been claimed that cold dark matter models do indeed show the effect (White et al., 1987; Park and Gott, 1990), but that is difficult to understand since we now know that the cold dark matter model is in fact deficient in large scale power.

It is significant that the one model which does seem to explain this observation is the Voronoi Clustering model (van de Weygaert, 1991). In that model the galaxy clusters are taken as being located at the vertices of a three-dimensional Voronoi tessellation, this provides the normalization for the model and so the model has no free parameters to juggle. The galaxies are then distributed on the faces of the Voronoi polyhedra and "observed" by drilling pencil-beams through the resultant distribution. The pair separation histograms show as much clustering as the Broadhurst et al. survey about one time in six.

It appears that pencil beams which start by going through the center of a void and almost perpendicular to the next wall have a good chance of going almost perpendicularly through the following wall. The cells of the Voronoi tessellation are highly correlated. However, many beams will intersect a wall at an angle such that the wall runs along the line of sight for a large distance, in which case there will be no periodicity observed.

What is lacking at the present moment is an objective series of statistical tests that will quantify the statistical significance of the pencil-beam data and objectively compare with model predictions. The situation is so serious that Kaiser and Peacock (1991) have argued that the apparent periodicity is not statistically significant and can be reproduced by a model in which galaxies are placed in randomly distributed clusters. Their argument is strong, but does not deny the possibility that the regularity is nevertheless real.

2.4 Surveys with Independent Distance Estimates

Having samples of galaxies for which there are distance estimates that are independent of the Hubble law is of crucial importance. It is true that such samples are necessarily considerably smaller than the redshift survey samples, numbering at most in the thousands of galaxies. There are of course strong selection effects in creating such samples. These arise out of the fact that properties of particular types of object are exploited to give the distance estimator. Thus there are different distance indicators for elliptical galaxies, for spiral galaxies in general, and for particular classes of spiral galaxy. The distance errors are generally very large (at least 20-30%) unless one focusses on a special type of galaxy of known luminosity (standard candles), and then there are hidden dangers as exemplified by the original Rubin-Ford sample of ScI galaxies (see section 2.4.1).

The analysis of such data sets is also non-trivial. One is tempted to fit models of clusters with power law halos, but that specific model fitting is fraught with dangers. What, for example, is the significance of a result obtained through a model that does not in fact represent the data? Bertschinger and Dekel have, in a series of important papers, described a method for reconstructing the full three dimensional distribution of galaxies and their flow relative to the cosmic background. Their technique, "POTENT", makes a plausible assumption about the nature of the velocity field that is being probed, and is potentially capable of giving us a good smoothed picture of what is going on in the Universe.

I shall discuss some data samples that have independent distance estimators, and then go on to discuss the Bertschinger-Dekel technique.

2.4.1 The Rubin-Ford Effect

The pioneering work of Rubin et al. (1976a,b) used a sample of ScI galaxies arguing that these were good "standard candles" whose true distances could be estimated from their apparent brightness alone. Their catalogue of 18 galaxies was pruned to reduce various biases to a sample of 96 objects having radial velocities in the range 3500 km s^{-1} to 6500 km s^{-1}. On analyzing the distribution of "true distance" relative to Hubble flow distance they found substantial motion of the Local Group of galaxies relative to their sample of distant ScI galaxies.

The Rubin et al. analysis yielded a Local Group mass center velocity of $V_{RF} = 454 \pm 125$ km s^{-1} towards $l = 163°$ and $b = -11°$ relative to the ScI sample. The Microwave Background Radiation dipole anisotropy implies a motion of the mass center of the Local Group of $V_{MWB} = 610 \pm 50$ km s^{-1} towards $l = 265°$ and $b = 480°$ relative to the cosmic frame (Smoot et al., 1991). These numbers take account of the motion of the Sun relative to the mass center of the Local Group $V = 295$ km s^{-1} towards $l = 97°$ and $b = -6°$. The conclusion is thus that the ScI galaxy sample as a whole is moving with velocity $V = 885$ km s^{-1} towards $l = 304°$ and $b = 26°$ relative to the frame of reference in which the microwave background radiation is isotropic.

The Rubin-Ford effect has been the subject of intense discussion ever since it was reported (Fall and Jones, 1976; Rart and Davies, 1982; Collins, Joseph and Robertson,1986; James, Joseph and Collins, 1991). I still think that this particular data set is indeed biased in the way described by Fall and Jones and the apparent large scale flow implied by that data is spurious. This particular bias arises only in samples of galaxies selected in a narrow range of absolute magnitudes, such as ScI galaxies and so one should be careful before arguing that such effects arise in other samples.

2.4.2 Samples of Elliptical Galaxies

For determining redshift independent distances to elliptical galaxies one can use a Faber-Jackson (Faber and Jackson, 1976) type relation between the isophotal diameter $D_{\rm n}$ of the galaxy where the surface brightness falls to some particular value and central velocity dispersion, σ_0. The relationship, $D_{\rm n} \propto \sigma^{4/3}$ which was found by Dressler et al. (1987) and Lynden-Bell et al. (1988) has been the subject of much discussion (Djorgovski and Davies, 1987; Lucey and Carter, 1988; de Carvalho and Djorgovski, 1989).

Lynden-Bell et al. (1988) (generally referred to as "S^7") have applied this distance indicator to a sample of ~ 400 elliptical galaxies with the rms depth of 6,000 km s^{-1} and find a large peculiar velocity of 600 ± 100 km s^{-1} on a scale of $\sim 50h^{-1}$ Mpc. The direction of this velocity vector is towards the Hydra-Centaurus system. This direction roughly coincides with the microwave background dipole direction, the dipole determined from spiral galaxy samples and the optical light dipole direction (Lahav, 1987). It also coincides roughly with the long axis of the quadrupole component of the local velocity field (Lilje, Yahil and Jones, 1986).

The discovery of bulk motions relative to the cosmic frame provided by the microwave background radiation, and of a coherent infall towards the direction of the Hydra-Centaurus part of the sky is of considerable importance. We discuss these in later sections.

A word of caution should be in order here: the new distance indicator was established by using only elliptical galaxies in the Coma cluster of galaxies. One knows, and generally expects, galaxy properties to be influenced by their environments (tidal interactions, mergers, gas removal etc. — see the review of Dressler (1984)). One cannot be sure at this stage whether the ($D_{\rm n}, \sigma$ relation applies equally to elliptical galaxies in other environments. Of the $\sim$ 400 ellipticals in the S^7 sample, a third are in rich clusters, a third in poor ones and a third in the field. However the sample is not large enough to estimate the contribution from these environmental effects or other likely evolutionary effects (Djorgovski, de Carvalho and Han, 1988; Silk, 1989). Much of the future discussion will turn around the quality of the distance indicator for elliptical galaxies.

2.4.3 Samples of spiral galaxies

For disk galaxies there is the Tully-Fisher relationship (Tully and Fisher, 1977) between the total luminosity and the asymptotic rotational velocity width of the 21 cm. HI line. This has been applied at a variety of wavelengths from the blue to the infrared (Aaronson et al., 1986; Stavely-Smith and Davies, 1989).

These samples of spiral galaxies have been used for a variety of purposes, though in general because of the inaccuracy of the distance estimator the data is best smoothed over relatively large volumes. (See Hesslbjerg-Christiansen (1991) for a potentially important way of improving these distance estimates). They have been used to rederive the motion of the local sample of galaxies relative to the microwave background, and to determine the quadrupole distortion of that flow (Lilje et al., 1986; Stavely-Smith and Davies, 1989). They have also been used to map out the motions of galaxy clusters relative to one another, since good estimates of distances to clusters can be obtained by averaging distance estimates for a number of cluster members.

The relative motions of clusters of galaxies was studied by Aaronson et al. (1986, 1989) by determining redshift independent distances to individiual member galaxies in some 11 clusters. They recovered a large scale flow of the Local Supercluster towards the direction that is now identified with the Great Attractor. What seems significant about their result is that the Hubble flow deviations are relatively small ($\simeq$ 300 km s^{-1}) when measured from clusters of galaxies.

A recent detailed discussion by Lucey et al. (1991) using the elliptical galaxies in dusters (and the $D_{\rm n} - \sigma$ elliptical galaxy distance indicator) confirms that the Hubble flow deviations for galaxy clusters are generally small, but that there are a few outstanding cases where there is an indication of substantial non-Hubble motion, particularly the cluster A2634. However, the authors comment that tidal stripping among galaxies in the central regions of this cluster may have been responsible for the apparent non-Hubble component of the flow.

2.4.4 The "Real" 3-Dimensional Distribution

Early studies of non-Hubble motions used fitted specific models for the Great Attractor and its environment (Lynden-Bell et al. 1988). While such models give an indication of what the Great Attractor is, one is left with a very large parameter space of possible models none of which has an a priori dynamical justification.

This model-fitting situation has been dramatically improved by the recent discovery of Bertschinger and Dekel (1989) that one could, on the basis of a few reasonable assumptions, reconstruct the entire three dimensional velocity field given only the radial peculiar velocity data for a sample of galaxies. Moreover, the sample does not have to be a complete sample (though where there are most galaxies the reconstruction of the cosmic flow field is obviously most reliable). Bertschinger, Dekel, Dressler and Faber (1991) in a recent series of papers, have applied the technique to a compendium of redshift samples that allow the universe to be mapped out to a distance of 6000 km s^{-1}.

The actual argument describing how to do this is quite complex, but it can be simplified for didactic purposes by taking a liberty with the coordinate systems being used.

Given a galaxy with radial velocity cz and velocity independent distance estimate r, the peculiar radial velocity is

$$V_{\rm r} = cz - H_0 r. \tag{44}$$

If we suppose that $V_{\rm r}$ is the radial component of a vector field $\mathbf{V}$ that is the gradient of a potential Φ, we can write

$$\mathbf{V} = -\nabla_{\mathbf{r}}\Phi(\mathbf{r}), \tag{45}$$

and this has solution

$$\Phi(\mathbf{r}) - \Phi(\mathbf{O}) = \int_{\mathbf{O}}^{\mathbf{r}} \mathbf{V}\cdot\mathbf{dl},$$

where O represents the observer (us). The integral can be taken over any path from $\mathbf{O}$ to $\mathbf{r}$, and in particular a radial path. This particular choice of path involves only the *radial* component of the velocity, which we know. In $\mathbf{r}$-space spherical polar coordinates (r, θ, ϕ):

$$\Phi(\mathbf{r}) = \int_0^r V_{\rm r}(r', \theta, \phi) dr'. \qquad (46)$$

We have set the potential equal to zero at the origin since we don't need its value, only its derivatives. Having got Φ at all points we can then determine the *three-dimensional* velocity field from it by doing

$$\mathbf{V} = -\nabla_{\mathbf{r}} \Phi(\mathbf{r}). \qquad (47)$$

The projection of this velocity along the line of sight is the contribution of the peculiar velocity to the observed recession velocity. Thus we can improve our estimate of the true distance to the galaxy.

We seem to have got something for nothing! In fact it was not for free. The price we had to pay was the assumption that the velocity field was derivable from a potential. That is why the method is called "potent".

2.5 Clusters and Voids

2.5.1 Galaxy Clusters

Rich galaxy clusters are prominent features on sky survey plates, but objects like the Coma cluster of galaxies are rather rare. According to Bahcall and Soneira (1983) the density of richness $R > 0$ clusters is $n_{R>0} = 7.5 \times 10^{-7}$ Mpc^{-3}. Abell (1958) catalogued these, and that catalogue has since been extended to the southern sky by Abell, Corwin and Olowin (1989). These clusters are selected in projection and it is rather difficult to assess the selection effects that go into making up such catalogues. Redshifts are now available for large numbers of Abell clusters (see, for example, Huchra et al., 1990).

Searching for clusters in three dimensional redshift surveys of galaxies was initiated by Geller and Huchra (1983) and then refined by using N-body simulations by Nolthenius and White (1987) and by Moore (1991). Such objective catalogues are very useful in a number of respects, especially when looking at the group/cluster multiplicity or luminosity function, or when calculating a cluster-cluster correlation function.

Moore (1991) calculates the distribution of cluster luminosities rather than the multiplicity function. The cluster luminosities can be related to their masses through an assumed Mass to Light ratio, and thence directly to theories for the origin of large scale structure.

Moore's main result is that the distribution of cluster luminosities derived from the CfA catalogue is best fit by

$$\phi(L)dL = \phi^{\dagger}\left[\left(\frac{L}{L_{\dagger}}\right)^{F} + \left(\frac{L}{L_{\dagger}}\right)^{B}\right]^{-1}\frac{dL}{L_{\dagger}}$$

where $F = 1.16 \pm 0.07$ is the faint end slope, $B = 1.7 \pm 0.12$ is the bright end slope, and $L_{\dagger}$ is a characteristic luminosity corresponding to absolute magnitude $M_{\dagger} = -22.2 \pm 0.2$. The normalization is given by $\phi_{\dagger} = 3.0 \pm 0.5 \times 10^{-4}$ Mpc^{-3}. At the end, a standard Schechter-Type luminosity function would be far too steep.

Another interesting aspect of galaxy clusters is the distribution of velocity dispersions. The Abell clusters show a rather flat distribution: $n \propto \exp(-3V_{1000})$, where V_{1000} is the cluster velocity dispersion on units of 1000 km s^{-1}, showing the existence of a substantial number of clusters with velocity dispersion in excess of 1000 km s^{-1} (Frenk et al, 1990). These authors suggested that the high velocity dispersions could be due to contamination by nonmember galaxies or superposed clusters. This is a claim that will only be resolved when we have a ROSAT generated galaxy cluster catalogue. The groups identified by Moore (1991) in the CfA catalogue have a much steeper distribution of velocity dispersions $n \propto \exp(-15V_{1000})$. The two distributions are equal around $V_{1000} = 600$ km s^{-1}. Most of the groups found in the CfA catalogue would not qualify as Abell clusters, they are not dense enough. What is interesting is that the CDM predictions for a bias parameter $b = 2 - 2.5$ produce reasonable agreement with the CfA group data.

As a final comment on clusters of galaxies, it should be remarked that the original de Lapparent slice contained an unusually large number of Abell clusters (A2162, A1267, A1185, A1213 and A617 besides A1656, the Coma cluster). Thus the CfA slice is rather special in this respect, especially in comparison with equivalent southern hemisphere slices (da Costa, 1991).

2.5.2 The Cluster-Cluster Correlation Function

The cluster-cluster correlation function (Klypin and Kopylov, 1983; Bahcall and Soneira, 1983; Bahcall, 1988a,b) is a power law falling to unity on scales $r_0 \approx 25h^{-1}$ Mpc. and remaining positive beyond $50h^{-1}$ Mpc. Taken at face value, this provides a strong argument against CDM models. Not only does CDM fail to predict such a large correlation length, r_0, the correlation length in CDM is depends only on the shape of the power spectrum and not its amplitude and hence the cluster-cluster correlation length is independent of the value of the bias parameter. This last point makes it difficult to fix CDM without saying the data has been wrongly interpreted.

Both Soltan (1988) and Sutherland (1988) have found evidence for strong anisotropies in the correlation function of clusters looked at in redshift space. They argued that projection effects enhance the apparent richness of galaxy clusters and so conspire to boost $\xi_{\rm cc}$. The real lengthscale according to these critics is $r_0 \approx 14h^{-1}$ Mpc. A simple model for this was presented by Dekel et al. (1989a,b), though the later study of the phenomenon by Sutherland and Efstathiou (1991) shows that the reasons are not so simple.

The anisotropies reported by Soltan and by Sutherland amount to peculiar velocities ~ 2000 km s^{-1}. It is difficult to see how they could be accounted for entirely by the relative motions of galaxies clusters as supposed by Bahcall (1988) and her collaborators since Hubble flow deviations as large as ~ 600 km s^{-1} are rare (Lucey and Carter, 1988).

There appears to be a trend in the amplitude of the cluster-cluster correlation function with cluster richness in the sense that the richest clusters have the greatest correlation lengths (see for example Bahcall and Soneira, 1983 and Bahcall, 1988). However, groups of galaxies selected from the CfA catalogue (Nolthenius and White, 1987; Moore, 1991) have a correlation function that agrees in amplitude and slope with the predictions of CDM. Given that the prediction of CDM is independent of bias parameter, and that the group selection algorithm seems to be quite effective (and objective), this must be regarded as a plus point for CDM. The disease could well be in the Abell cluster catalogue and we must await surveys based on the ROSAT satellite to give an alternative rich galaxy cluster catalogue.

2.5.3 Voids

The Bootes void (Kirshner et al., 1981) with diameter $\sim 60h^{-1}$ Mpc was the first void to attract attention. Originally it was thought to be totally devoid of galaxies. Later observations did find some galaxies there (Thuan et al., 1987; Dey et al., 1989), but the region is still well underdense for its size. The latest count is 21 galaxies, of which 13 are IRAS sources, in a region where one expects to find 33 to 40 galaxies

(Dey et al., 1989) making the density contrast there $-5/6 \leq \delta \leq -2/3$ (an underdensity by a factor 3 to 6).

The de Lapparent slice and other redshift catalogues revealed voids in abundance, giving the impression that the structure of the universe is bubble-like and entirely dominated by the voids and their walls. The bubble-like appearance may be an artifact of peculiar velocities distorting the real map of galaxy distribution (Kaiser, 1988). However, the velocities are unlikely to significantly reduce the (large) size of typical voids which should be explained by any successful theory for formation of the large-scale structure in the Universe. Nor are the voids a consequence of extreme luminosity segregation (Dekel and Silk, 1986): the distribution of low surface brightness galaxies (Thuan et al., 1987, 1991) and of IRAS galaxies (Babul and Postman, 1990) follows that of bright ones.

Einasto, Einasto and Gramman (1989) have studied a compilation of voids, and found that their mean radius is $\sim 50h^{-1}$ Mpc. However, more recently, Kauffmann and Fairall (1991) applied a void finding algorithm to various redshift catalogues and found that the distribution of void diameters for their void catalogue peaked in the range 8-11 h^{-1} Mpc., with a long tail in the distribution extending to voids the size of the Bootes void or larger. It is not known whether these voids are truly empty of material. Einasto et al. conclude, on the basis of their numerical simulations (which included a cosmological constant), that one third of the material in the universe could reside in the voids. The void radii are sensitive to the bias level applied to the simulation, so this may be a good way to fix the bias parameter b.

As a final comment, it should be noted that the distribution of rich Abell clusters also reveals large voids. There are two unusually large voids in the northern sky Abell catalogue (Bahcall and Soneira, 1983; Huchra et al., 1990). This is interesting in relation to the Voronoi clustering model for Abell clusters (van de Weygaert, 1991) which purports to provide an explanation for the regularity in the redshift distribution of galaxies in pencil beam surveys (Broadhurst et al., 1990) since that model is normalized relative to the rich cluster distribution. The voids in the Voronoi model are on average $125h^{-1}$ Mpc. across and these presumably correspond to the voids seen in the Abell catalogue.

2.6 Great Attractors, Dipoles and all that

2.6.1 The Great Attractor and Others

Lynden-Bell et al. (1988) identified a large scale enhancement in the distribution of galaxies as being a possible cause of the large scale Hubble flow deviations discovered in the S^7 survey of elliptical galaxies. They dubbed this "the Great Attractor".

The remarkable flows in the direction of the Great Attractor, as deduced from the elliptical galaxy survey, receive some support from a survey of spirals for which distances have been determined using the IR Tully-Fisher distance indicator (Aaronson et al., 1989).

The existence of this attractor had been guessed at already by Lilje, Yahil and Jones (1986) who identified a quadrupole component in the Virgocentric flow on the basis of the Aaronson et al. (1982) survey of the Virgocentric flow. The long axis of the quadrupole might have been expected to point towards the center of the Virgo cluster, but it was found to point in the direction of the Hydra-Centaurus complex. The quadrupole has since been confirmed by the independent data set of Stavely-Smith and Davies (1989). The importance of the quadrupole component is that it is unambiguously gravitational in origin. The quadrupole component of the force exerted by neighboring masses falls if as r^{-3} and so the quadrupole imposes constraints as to where the mass causing the distortion of the flow is located.

The Great Attractor can be modelled by density distribution $\rho \propto r^{-2}$ centered about a point 3,500 km s^{-1} away from us toward the Hydra Centaurus region with the total mass of $\sim 10^{16} M_{\odot}$. The detailed interpretation of the Local Group motion relative to the MWB is analyzed by Lynden-Bell, Lahav and Burstein (1989). There are local contributions to Local group motion as well as contributions from the directions of Perseus and Hydra-Centaurus. See the discussion on the "POTENT" method of reconstructing the cosmic density distribution and peculiar velocity fields.

Because the attractor lies in the Galactic plane, it is difficult to map. The search for the attractor lead other groups (Scaramella et al., 1989; Lahav et al., 1989; Raychaudhuri, 1990) to the discovery of a "super attractor" far beyond the Hydra-Centaurus system, ($\sim 140h^{-1}$ Mpc.) having about ten times the size of the proposed 'Great Attractor". This super attractor is a strong concentration of rich clusters of galaxies. Its distance is, however, so great that it is unlikely to be responsible for the tidal forces that lead to the quadrupole distortion of the Virgocentric flow.

The existence of a Great Attractor poses a number of problems for theories of the origin of large scale structure. Bertschinger and Juskiewicz (1988), for example show that the Great Attractor is a 7σ fluctuation in CDM models. Playing with the bias factor in CDM models can solve the problem, but creates other problems. It seems that only Peebles' (1987) isocurvature baryonic models with $\Omega_0 = 0.4$ and a very flat power spectrum ($n = -1$) can solve the problem.

2.6.2 Dipole Convergence

The Local Group, and its surroundings are moving at what by cosmological standards is a high speed relative to the cosmic frame defined by the microwave background radiation. Although Rubin, Ford and collaborators (Rubin and Ford, 1976a,b; see section 2.4.1) had reported a high velocity of the Local Group relative to a sample of distant galaxies, the first unequivocal detection of the motion came from the microwave background dipole anisotropy (Smoot, Gorenstein and Muller, 1977; Lubin et al., 1983, 1985; Fixen et al, 1983). The COBE satellite's Differential Microwave Radiometers currently measures a dipole with an amplitude of 3.3 ± 0.2 mK pointing in a direction towards $l = 265^\circ \pm 2^\circ, b = 48^\circ \pm 2^\circ$ (Smoot et al, 1991).

What was surprising was that the motion inferred from the microwave background dipole did not point in the direction suggested by Rubin et al. (1976a,b), and nor did it point in the 'natural' direction, towards the Virgo Cluster of galaxies, our nearest significant mass concentration. There was indeed a significant component towards the Virgo cluster and this was viewed as a part of the general Virgocentric Infall (Davis and Peebles, 1983). The source of the main component of the motion was not apparent until the discovery by Lynden-Bell et al. (1988) of a systemic motion of the Local Group towards what has become known as the Great Attractor.

I shall discuss the nature of the Great Attractor elsewhere in these lectures, all that should concern us here is the detection of the same motion relative both to a sample of Elliptical Galaxies and relative to the cosmic frame in which the microwave background radiation is isotropic. That motion has been confirmed in many subsequent surveys using different types of objects: spiral galaxies, Abell clusters, optically selected galaxies, ... Note that it is not only the Local Group that is moving, but at least the local neighbourhood, as is evidenced by the quietness of the local Hubble flow (Brown and Peebles, 1988). Also, the fact that the long axis of the quadrupole distortion of the flow of galaxies in the Virgo supercluster points in the same direction is also of importance (Lilje et al., 1986; Stavely-Smith and Davies, 1989).

It is an important goal of cosmology to reconstruct the entire local flow from the distribution of known material in the universe. The question is where is the matter that is exerting this pull on the Local Group and its environs? The answer of Lynden-Bell et al. (1988) was the Great Attractor, a vast aggregation of galaxies largely hidden from our view by the Galactic plane. The attempts to confirm this hypothesis have met with some difficulty. We see for example the claim of Rowan-Robinson et al. (1990) that the entire effect can be explained in terms of known systems of galaxies, and that there is no need to invoke any special unseen objects like the suggested Great Attractor. There are also claims that there is a detectable influence on the Local Group motion from distances far beyond the Great Attractor (Plionis and Valdarmini, 1990).

The issue is referred to as *dipole convergence*. As the influence of galaxies from ever larger shells is counted, the direction of motion of the Local Group should converge in magnitude and direction to the microwave back ground dipole direction. Several corrections have to be made, the most important being a correction for the galaxies and clusters that are hidden from sight because of the Galactic Plane. There is another correction that has to be made: we measure the microwave background anisotropy from the point of view of the Solar System frame of reference whereas the dynamical forces act on the Local Group mass center and on the Galaxy. A failure to achieve convergence could mean one of several things. It could mean that the light distribution does not trace the mass, it could mean that we have not gone far enough in distance, or it could simply be that we do not know the motion of the Sun relative to the mass center of the Local Group.

If the distances to the galaxies are known, we can sum up the contributions from shells of ever increasing size to the local force field. Note that in order to fit the amplitude of the dipole, we must assume a value of $\Omega_0/b^{5/3}$. The bias parameter b comes in because we look at the fluctuations in the light distribution, not the mass distribution.

Light and gravity both fall of as R^{-2}, and so, if light traces mass, a dipole in the gravitational force field should show up as a dipole in the light distribution. Thus for a sample of objects drawn from a catalogue, we can deduce the contributions of various parts of the Universe to the local force simply by examining the distribution of the light and making the necessary assumption that light traces mass. This has been done for virtually every available catalogue of galaxies: spiral galaxy samples, elliptical galaxy samples, the IRAS-based samples and for samples of galaxy clusters.

Lahav (1988) summed up the light in diameter limited subsamples of a catalog of optically identified galaxies as a function of limiting optical diameter He found a light dipole in roughly the expected place, though he found convergence in direction was achieved at a relatively nearby distance. An enhancement in the galaxy density in the direction of the Great Attractor is clearly seen in his plots of the distribution of galaxies on the sky. This has also been done for the IRAS survey (Strauss et al., 1988; Yahil, 1988), and for the QDOT survey (Rowan-Robinson et al., 1990), and it is clear that convergence in direction is being achieved, though the dipole direction does move around on the sky quite a lot data from ever large shell is considered.

Plionis and Valdarmini (1991) study the dipole contribution from all galaxy clusters in the Abell-ACO catalogues having their 10^{th} brightest galaxy brighter than in $m_{10} = 16.4$. The catalogue is 80% redshift complete. They calculate the dipole moment of the light distribution from the clusters as a function of depth in their catalogue, using the population of each cluster as an estimator of the total cluster light. In order to calculate the acceleration due to a given cluster, they rescale the light cluster light to a mass in such a way that they get the correct answer for the Coma cluster mass. Most of the effect comes, as expected, from a volume of radius $r < 50h^{-1}$ Mpc., but there still appears to be a substantial contribution from the Shapley concentration of galaxy clusters and clusters in general out to the limit of their survey. What is perhaps surprising about this result is that the clusters should trace the mass distribution so well. Only a few percent of all galaxies are in such clusters and it would have been quite conceivable that the contribution to the local gravitational field from the clusters would be swamped by the contributions from all the other galaxies.

Recall the discussion of the QDOT survey and the fact that Rowan-Robinson et al. (1990) were able to obtain satisfactory dipole convergence using only clusters that had been identified in the QDOT survey. There was no need for 'extra' mass hidden behind the Galactic plane.

Rowan-Robinson et al. refer to highly extended (and overlapping) cluster halos which have power law density distributions ($\rho \propto r^{-1.6}$) extending out to $30h^{-1}$ Mpc. Only 1.5% of the cluster mass lies within the Abell radius! It is presumably these overlapping halos that the POTENT reconstruction is finding and labelling the Great Attractor with a density contrast of around 0.7.

The outstanding question is at what distance is convergence in direction achieved? The amplitude of the velocity can be fixed by selecting a mass to light ratio for the objects in the sample, and that is equivalent to fixing the parameter $\Omega_0/b^{0.6}$ with a bias parameter, b, appropriate for the sample. There are several aspects to this question. Firstly there is the question of the statistical significance of the inferred direction and the value of the velocity. There is then the question as to whether such an amplitude is expected in a given theory and the confidence with which we can determine Ω_0 and b.

Several people (Kaiser and Lahav, 1089; Juskiewicz, Vittorio and Wyse, 1990; Lahav, Kaiser and Hoffman, 1989) have constructed models for Local Group peculiar velocity, v_R, in Cold Dark Matter and Baryonic Dark Matter models. The data from the IRAS and elliptical galaxy surveys out to 10,000 km s^{-1} cannot tell the difference between these models with any confidence. Another approach (Regos and Szalay, 1989) is to use multipole expansions of the observed radial velocities in shells to shed light on the uncertainties in deriving the bulk velocity vector. It seems that distinguishing between what observers measure and what theorists talk about is a large part of the problem.

It would be important if there were a difference between the true peculiar velocity of the Local Group, $\mathbf{v}$ as inferred from the MWB dipole and the estimate $\mathbf{v}_R$ (as inferred from the volume averaged motion in a sample of galaxies (Vittorio and Juskiewicz, 1987)). One of the uncertainties in converting the motion of the Solar System relative to the microwave background to the motion of the mass center of the Local Group relative to the cosmic frame is the unknown dynamics of our Galaxy relative to the Local Group mass center. There is even a contribution from the rotation of the Local Group which is only poorly determined (Moore, 1991).

2.6.3 Large Scale Flows and CBR Anisotropy

It is an important question as to whether these non-Hubble motions arose from density fluctuations that were consistent with observed limits on the anisotropy of the microwave background radiation temperature. The anisotropy on such scales results simply from the Sachs-Wolfe effect.

Juskiewicz, Górski and Silk (1987) and Martínez-González and Sanz (1989) calculate the minimal microwave background anisotropy associated with a given streaming motion. By 'minimal' what is meant is that the density fluctuation spectrum is chosen so as to minimize the observed temperature fluctuations, subject to the constraint that they can also produce the velocity field. Although it is a near thing, no microwave background anisotropy experiments are yet in conflict with the observed non-Hubble motions.

Of course, it might be that the actual spectrum gives an observable temperature fluctuation. Doroshkevich and Klypin (1988) used the Zeldovich approximation to describe the evolution of velocity correlations on very large scales, and they also calculated the expected temperature anisotropies for the purposes of comparison with the RELICT experiment. They argued in favour of needing a feature in the spectrum of fluctuations on scale of 50-100 h^{-1} Mpc.

2.7 Velocity Correlations

There has been much discussion on the use of the velocity correlation function (Peebles, 1987; Kaiser, 1988):

$$\xi_v(r) = \langle \mathrm{v}(r) \cdot \mathrm{v}(0) \rangle = \frac{\Omega_0^{1.2}}{2\pi^2} \int_0^\infty P(k) j_0(kr) dk. \tag{48}$$

It is on the basis of this function calculated for the S^7 sample that Groth, Juskiewicz and Ostriker (1989) argue that the observed velocity field is far more correlated than would be expected on the basis of the CDM model, particularly at large separations.

However, Górski et al. (1989) have made an improvement on this and show how to calculate velocity correlations from observable quantities. They split the velocity correlation tensor into "parallel" and "transverse" components. They show, on the basis of comparison with cosmological N-body simulations, that the scalar version of this by Groth et al. (1989) may give misleading results. The Aaronson et al.

and IRAS samples show velocity correlations over scales of 1500 h^{-1} Mpc. The Burstein et al. elliptical galaxy sample shows the same correlation length scale, but with a considerably greater amplitude. (The elliptical galaxy sample correlations are very sensitive to the velocities of a small number of galaxies in the sample). The correlations in the Aaronson et al. (1982) and IRAS samples are not unusual for CDM models with a bias parameter $b = 2$ or less, but the CDM models cannot reproduce amplitude of the elliptical galaxy sample. The Peebles baryonic model with isocurvature fluctuation automatically generates long range correlations, but even then cannot come to terms with the S^7 elliptical sample.

3 CLUSTERING MEASURES

3.1 Two-Point Correlation Functions

I have reviewed the two-point correlation function $\xi(r)$ earlier (section 2.1 The point to recall here is that the large scale structure, on scales in excess of $20h^{-1}$ Mpc., is not described by measurements of $\xi(r)$. There is too much noise on these scales to even say whether $\xi(r)$ is positive or negative. It is also worth recalling that despite what our eyes tell us, the structure on these scales is indeed linear and of small amplitude. That can be seen by smoothing the galaxy distribution with a sphere of this scale or greater. We know in fact that the variance of the optical galaxy counts averaged over spheres of $8h^{-1}$ Mpc. (ie. 800 km s^{-1}) is unity, and this fact provides us with one basis for normalizing N-body experiments (given a bias parameter).

So why do we see all that impressive structure? The reason is that we are looking at the combined effects of large amplitude fluctuations on small scales that are correlated (albeit weakly) on large scales. The walls that define the voids are seen by virtue of their small scale structure (without which they would not look like walls!) and they look like walls or filaments because of the way they are organised on the larger scales. If we could measure the two point function reliably on the large scales we would see evidence of this.

So there is some motivation to look for clustering descriptors that quantify what our eyes tell us: that there is organised large scale structure. Note that because the structures we are seeking to quantify are linear, they may have no special dynamical significance. The Great Wall did not arise because sphere of that size collapsed to make a pancake! Not only is that dynamically unreasonable, it would also conflict with the isotropy of the microwave background radiation. Similarly, the fact that we see a great void does not imply that some region exploded from a small volume to form that space and its surrounding walls. Making that assumption would again lead to a problem with the isotropy of the microwave background (Barrow and Coles, 1990).

3.2 Higher Order Correlation Functions

Since the two point function alone does not describe the large scale structure, the idea is that the higher functions may fill in the gap. The three- and four- point functions have been extensively discussed and measurements of these functions has given rise to the idea that all high order correlation functions are sums of multiples of the two point function. The three point function for example (Peebles and Groth, 1975; LSSU) can be written as

$$\begin{aligned} \zeta_{123} &\simeq Q(\xi_{31}\xi_{12} + \xi_{12}\xi_{23} + \xi_{23}\xi_{31}) \\ Q &\simeq 1, \qquad \text{for} \qquad r < 2h^{-1}\ \text{Mpc} \end{aligned} \tag{49}$$

In this notation 1,2,3 denote the positions of the members of triples of galaxies. $\zeta(123)$ is to be thought of as a function of triangles of various specific kinds specified by the lengths of their sides (12), (23), (31). Then the scaling of ζ can be verified for all similar triangles of a given type, specified only by their size. Or it can be verified for all triangles in which two sides are the same as a function of the length of the third side.

It is worth noting the absence of a term proportional to $\xi_{12}\xi_{23}\xi_{31}$. The argument is that this term would dominate as $r \to 0$ and we do not see that happening on the scales where the three point function has been determined.

The 3-point function plays a role in the *Cosmic Virial Theorem* which relates the mean square peculiar velocity $\langle v_{21}^2(r)\rangle$ on scale r to the clustering as measured by the 3-point function (LSSU section 75, Peebles (1980)):

$$\langle v_{21}^2(r)\rangle = \frac{6G\Omega_0\rho_c}{b^2\xi(r)} \int_r^\infty \frac{dr}{r} \int \frac{\mathbf{r}\cdot\mathbf{z}}{z^3}\, \zeta(r, z, |\mathbf{r}-\mathbf{z}|d^3z. \tag{50}$$

This uses the (unverified) assumption that the distribution of $\mathbf{v}_{21}$ is isotropic. The bias parameter b comes in because the Cosmic Virial

Theorem involves the correlation functions of the mass distribution, and these are estimated from the distribution of light. Using a model for the 2-point function

$$\xi(r) = \left(\frac{r_0}{r}\right)^{\gamma}, \quad \gamma = 1.8, \quad r_0 = 4.1h^{-1}\ \mathrm{Mpc}.$$

and the relation between the 3-point and 2-point functions, we get

$$\langle v_{21}^2(r)\rangle = C_\gamma Q b^{-2}\Omega_0 H_0^2 r^{\gamma} r_0^{2-\gamma}, \tag{51}$$

where C_γ is a constant depending on γ. In this equation, the only unknown quantity is Ω_0.

Q has been determined for the Durham/AAT/SAAO redshift surveys (Hale-Sutton et al., 1989) to have the value

$$Q_{Durham} = 0.58 \pm 0.05, \tag{52}$$

from data on scales $r < 1$ Mpc. This is somewhat lower than the value $Q = 1.3 \pm 0.2$ from the analysis of projected data by Groth and Peebles (1977). With this value of Q and the velocity dispersions measured from their survey they find

$$\frac{\Omega_0^{Durham}}{b^2} = 0.18 \pm 0.09, \tag{53}$$

again using fits for $r < 1$ Mpc. The error bar seems somewhat conservative since the variation of Ω_0 determinations from the subsamples that make up the Durham/AAT/SAAO survey is quite considerable. The value is nonetheless on the low side if one is aiming at $\Omega_0 = 1$. The value only reflects the clustered mass on scales < 1 Mpc., and there is some freedom in choosing the bias parameter.

Coles and Jones (1991) point out that Q may not be the best quantity to measure departures from Gaussian behaviour, particularly on scales where correlations are weak. They suggest instead the direct measure of the skewness:

$$\Gamma = \left[\frac{\zeta^2}{\xi^3}\right]^{\frac{1}{2}}. \tag{54}$$

For a Gaussian random distribution of galaxies, Γ will be zero. In general, Γ will depend on scale and the shape of triangles used to measure ζ.

Even higher order correlation functions can be used to measure the clustering (Sharp, Bonometto and Lucchin, 1984), but despite clever tricks they are difficult to measure (Szapudi, Szalay, and Bascan, 1991) and lack any intuitive appeal. (See also Jones and Coles (1991) for more comments on the 4-point correlation function and its relationship to the kurtosis of the underlying distribution).

3.3 Counts in cells

The counts of galaxies in cells are related to the correlation functions of all orders and potentially provide an important means of testing for the presence of voids in a sample of galaxies. The relationship between the probability $P_N(V)$ of finding N galaxies in a sample volume V and the correlation functions of all orders was given by White (1979). The particular case $P_0(V)$ is called the *Void Probability Function*, 'VPF' for short, and is thought to be a sensitive discriminator of clustering models.

The probability that a volume V, randomly selected in a sample of points having mean number density n_0, will contain no galaxies was first given by White (1979)

$$P_0(V; n_0) = e^{-an_0 V}. \tag{55}$$

P_0 depends on the mean density of the sample, and in fact it can only depend on the product $n_0 V$. The scale a is given in terms of the correlation functions of the distribution:

$$a = 1 + \sum_{i=1}^{\infty} (-n_0)^{i-1} \int \xi_i dV_1 \ldots dV_{i-1}. \tag{56}$$

Here ξ_i is the i-point correlation function of $(i-1)$ coordinates and is determined on linear scales by (among other things) the power spectrum of the primordial density fluctuations. For purely Gaussian fluctuations the sum in a is cut off beyond the second term, but as we discussed in the section on correlation function, gravitational evolution destroys the Gaussian character of fluctuations. If we wish to compute $P_0(V)$ in a general case we are forced to make an ansatz about the relationship between second and higher order correlation functions either through BBGKY hierarchies or by intelligent guesswork (Schaeffer, 1985; Fry 1986). The data can then be used to test this hypothesis. The VPF was first studied observationally by Maurogordato and Lachieze-Rey (1987) who were able to confirm the Schaeffer scaling relations. The recent article by Einasto et al. (1991) provides a clear exposition of what the Void probability function actually measures. Cappi, Maurogodato and Lachieze-Rey (1991) have confirmed that the VPF of the distribution of rich galaxy clusters shows scaling behaviour up to a scale of 50 h^{-1} Mpc,.

$P_0(V; n_0)$ should be distinguished carefully from the probability of finding a void of the kind that has been identified as a feature of the large scale galaxy surveys. The VPF describes the probability that a randomly placed sphere of a given volume V contains a given number of galaxies — not the probability of finding a region of volume V which is devoid of galaxies.

The probability of finding N galaxies in a randomly selected volume $V, P_N(V)$ has been discussed in terms of quite general scaling hypotheses by Balian and Schaeffer (1989a,b). Balian and Schaeffer were able to compute the properties of the counts-in-cells distribution $P_N(V)$ on the hypothesis that the higher order correlation functions are related to the two-point correlation function through rather general scaling hierarchies. The CfA survey data appears to support both the form of $P_N(V)$ and the Balian-Schaeffer scaling hypothesis (Maurogordato and Lachieze-Rey, 1987; Alimi, Blanchard and Schaeffer, 1990). There is an extensive analysis of galaxy counts in cells by Fry et al. (1989).

An alternative approach to the counts in cells distribution was taken by Saslaw and Hamilton (1984) who argued that

$$\begin{aligned} P_N(V) &= [(1-\beta)n_0V + \beta N]^{N-1} e^{-(1-\beta)n_0V - \beta N} \\ \beta &= 0.70 \pm 0.05. \end{aligned} \tag{57}$$

The value of the constant β (called b by Saslaw and Hamilton, but we wish to avoid confusion with the bias parameter) is from Crane and Saslaw's (1986) analysis of the Zwicky catalogue of galaxies. The parameter β is interpreted physically by Saslaw and Hamilton as being the ratio $\beta = -W/2K$ of the gravitational correlation energy, W, to the kinetic energy in peculiar motions,K. In fact, β could depend on scale and will certainly depend on time. This distribution function is discussed at length in Itoh et al. (1990a,b).

What is interesting is that this distribution function fits N-body models rather well (Suto, Itoh and Inagaki 1988, 1990), provided that β depends on scale as $1-\beta(r) \propto r^{-(3-\gamma)/2}, \gamma = 1.8$ being the slope of the two-point correlation function. Itoh (1990) has an interesting discussion of the relationship between the Saslaw-Hamilton distribution function and the fractal dimensions D_q of a set having this distribution function, though he somehow ends up with a set whose Hausdorff dimension D_0 is smaller than the correlation dimension D_2.

3.4 Genus

A promising way of classifying the structure of the universe is to see how the topology of the surfaces of constant density in the universe varies with the value of the density (Gott, Melott and Dickinson, 1986; Weinberg, Gott and Melott, 1987). The topological genus of the set of points enclosed by surfaces of constant density is defined as the number of holes minus the number of isolated regions (though this is not how it is calculated). Thus a toroidal region has a genus of -1 and an isolated sphere has a genus of 0. The genus signature of the distribution is calculated as a function of the density threshold at which the surfaces are defined. Several authors, starting with Doroshkevich (1970) calculated the genus signature for a windowed Gaussian random density field as a function of the threshold ν defining the surfaces:

$$G_\nu \propto (1-\nu^2)e^{-\nu^2/2}. \tag{58}$$

ν is measured in terms of the variance of the field seen through the selected window. The constant of proportionality depends on the power spectrum of the windowed density distribution. If the distribution is non-Gaussian, this would show up in the genus signature.

This has been discussed at length in two recent papers Gott et al. (1989) and Melott et al. (1989). The main technical problem is that of relating the observed two dimensional sections of three dimensional data to the three dimensional situation. In the data paper (Gott et al, 1989), catalogues of all kinds of objects ranging from dwarf galaxies to Abell clusters are used. It is found that the universe is "sponge-like" on largest scales, with no evidence for a bubble-like structure which, it is claimed, could have been detected if present. There are voids, but they are interconnected, as opposed to being surrounded on all sides by galaxies. On smaller scales, the topology has a tendency to become "meatball-like".

Moore (1991) has looked at the QDOT redshift survey, smoothed over a variety of windows. The genus signature is clear on scales of

$20h^{-1}$ Mpc. and $40h^{-1}$ Mpc. and is consistent with a Gaussian distribution of density fluctuations taken from a power law power spectrum with spectral index $n = -0.79 \pm 0.35$ on those scales. This appears to be a variance with the general hypothesis that the spectrum has the Harrison-Zel'dovich form on such scales, and in particular with the standard CDM models.

3.5 Multi-Fractals

The power-law nature of the two-point and higher order correlation functions on small scales is suggestive of some kind of scaling behaviour, at least in the range of scales where the power law is observed. The simplest structure that has such scaling properties is the simple fractal, first studied in this context by Efstathiou, Fall and Hogan (1979). Martínez and Jones (1990) have since shown that even in this apparent scaling regime, the distribution of galaxies is significantly more complex.

If the structure is not that of a simple fractal, what might it be? Jones et al (1988) have suggested that it may be *Multifractal* — that is, a distribution which over a certain range of scales can be characterized by a set of dimensions rather than just one dimension. There is scaling, albeit of a rather complex kind.

The multifractal description of the clustering process goes beyond the two-point correlation function, encapsulating all high order correlation functions in one function, D_q. The power of the technique has been demonstrated by Martínez et al. (1990) who examine the scaling structure of a number of well known clustering models and provide a variety of algorithms for calculating the dimensionality function D_q. The simplest of these is from the formulae for what is in fact the Renyi dimensions of the point set:

$$D_q = \lim_{r \to 0} \frac{1}{q-1} \frac{\log \sum_{i=1}^{N(r)} (n_i(r)/N)^q}{\log r}, \qquad q \neq 1, \tag{59}$$

$$D_1 = \lim_{q \to 1} D_q, \qquad q = 1.$$

Here, $n_i(r)$ is the count of particles in the ith cell of size r, and N is the total number of particles in all cells. There is a technical problem involved in taking the limit in a discrete sample as the cell size goes to 0.

In practise, box counting methods of determining dimensions are rather inefficient and tend to be dominated by shot noise. Van de Weygaert, Jones and Martínez (1991) have shown how to use the minimal spanning tree construct to calculate these dimensions with considerably fewer points than would be required by standard box counting methods.

The function D_q is related to the moment generating function, $m_q(r)$ of the clustering distribution. The relationship is

$$D_{q+1} = \lim_{r \to 0} \frac{1}{q} \frac{d \log m_q(r)}{d \log r}. \tag{60}$$

Hence each D_q encapsulates the information contained in the statistical moments of the distribution of the point set. The fact that in general one can in principle translate between the moments and the dimensions means that they contain the same information, though the information is presented in a different way. It is arguable that $D - q$ has a more immediate physical appeal.

The simplest application of the method is to compare the Hausdorf dimension, D_0, with the Correlation Dimension, D_2. Applying this to the ZCAT redshift catalogue gives values

$$\begin{aligned} D_0 &= 2.1 \pm 0.1 \\ D_2 &= 1.2 \pm 0.1. \end{aligned} \tag{61}$$

This leads to the strong conclusion that the universe is not a simple fractal characterized by one dimension. The value of D_0 indicates that the characteristic structures are sheet-like rather than filament-like. The value of D_2 is just $3 - \gamma$ where $\gamma = 1.8$ is the slope of the two-point correlation function.

4 THE INHOMOGENEOUS UNIVERSE — THEORY

4.1 The Origin of the Fluctuation Spectrum

We do not yet have an adequate understanding for the origin of the power spectrum of Primordial Density Fluctuations. Therefore it is usually assumed that primordial density fluctuations with an initial power spectrum $\mathcal{P}(k) \propto k^n$ were present at some initial time t_i either as part of the initial conditions of the Universe or having formed through some causal process which presumably would specify n and/or their amplitude. The case $n = +1$ is generally referred to as the Harrison-Zeldovich spectrum, after the individuals who advocate its use on the grounds that this was the power spectrum that did the least violence to the geometry of spacetime on any scale.

The "natural" Harrison-Zeldovich spectrum of fluctuations may not be so natural. In inflationary cosmologies the emergent spectrum of fluctuations depends on the details of the fields that cause the inflation (Kofman and Linde, 1987; Kofman and Pogosyan, 1988; Kofman and Blumenthal, 1989; Matarrese, Ortolan and Lucchin, 1989; Hodges, Blumenthal, Kofman and Primack, 1990; Salopek and Bond, 1991). These papers discuss specific models for the generation of power law spectra, spectra that are non-Gaussian and even non-power law spectra. The non-power law spectra are of interest partly because we see here a direct influence of the parameters of microscopic physics on large scale cosmic structure.

The issue is very contentious. For example, the Kofman and Linde (1987) idea that inflation driven by multiple scalar fields might create non-Gaussian fluctuations has been disputed by Hodges (1989). (See also Hodges and Blumenthal, 1990) In an analytically solvable case Barrow and Coles (1990) find that although non-Gaussian fluctuations can be generated, they become Gaussian as a state of exponential expansion is attained. Nevertheless, power law spectra with index different than 1 (Harrison-Zeldovich) can be created.

Peebles (1989) discusses the origin of isocurvature fluctuations in the inflationary scenarios. The question of the origin of isothermal perturbations as a consequence of the baryongenesis process at very early epochs is discussed by Barrow, Copeland, Kolb and Liddle (1991).

4.2 Modelling the Evolution of Large Scale Structure

4.2.1 CDM models

The most extensively studied spectrum of primordial density fluctuations is the one arising from the "Cold Dark Matter" theory in which the mass density of the universe is made up to $\Omega = 1$ by massive weakly interacting particles such as axions. The spectrum at recombination is entirely determined by the initial spectrum (generally taken to be Harrison-Zeldovich) and the physical processes that occur in the pre-recombination fireball. The bulk of the work on this model has been done numerically and is described in a series of papers by Davis, Efstathiou, Frenk and White (Efstathiou et al., 1985; Davis et al., 1985; White et al., 1987; Davis and Efstathiou, 1988).

Apart from the initial amplitude of the spectrum, which determines the epoch of galaxy formation, the theory is entirely fixed and has no free parameters as far as the dynamics is concerned. However, when it comes to relating mass and light distributions it is found necessary to introduce a *bias parameter*, b, defined in terms of the density contrasts of the distribution of matter and luminosity by

$$\left(\frac{\delta\rho}{\rho}\right)_{galaxies} = b\left(\frac{\delta\rho}{\rho}\right)_{matter}.$$

b enters into the normalization of the model. This universe has $\Omega_0 = 1$ and models are normalized so that the rms light fluctuation in a sphere of radius 800 km s^{-1} is unity. Hence the normalization of the mass fluctuations in 800 km s^{-1} spheres is just b^{-1}.

(This normalization is not the only possibility. It is possible to choose the scale where the two-point correlation function drops to unity to be $5h^{-1}$ Mpc, or to normalize through the function $J_3(r) = \int r^2\xi(r)dr$ whose (somewhat uncertain) value is estimated to be $J_3(10h^{-1}\text{ Mpc}) = 277h^{-3}\text{ Mpc}^3$ and $J_3(30h^{-1}\text{ Mpc}) \approx 800h^{-1}\text{ Mpc}^3$ (Davis and Peebles, 1983).)

There have been many theoretical discussions of the value of b (see Dekel and Rees, 1987, for a review), but there are no reliable methods of estimating what its value should be. At present b has to be inferred from the observations. Braun, Dekel and Shapiro (1988) looked into various biasing mechanisms and showed that it is possible to get galaxy formation to start at $z \sim 3$ and have a galaxy-galaxy correlation function with the correct slope today. (Though there was then a problem with the correlation length scale).

The value $b = 2.5$ is motivated by the N-body models Davis et al. (1985). However, $b = 2.5$ appears to be inconsistent with the observed streaming motions (Bertschinger and Juskiewicz, 1988; Górski, 1988). Kaiser (1988), for example, prefers $b = 1.5$. Peebles, Daly and Juskiewicz (1989) review this question in detail, looking at the consequences of various choices for b and for the lengthscale for mass clustering, r_0. They argue, for example, that $b = 1.5$ is too low to be consistent with the pairwise galaxy velocity correlation if $b = 1.5$ and $r_0 = 7h^{-1}$ Mpc. Lowering r_0 to 4 h^{-1} Mpc creates problems with cluster velocity dispersions (Evrard, 1989).

One way around these problems may be to introduce non-Gaussian initial conditions. Several numerical simulations have been done on these lines by Messina et al. (1990), using negative binomial and lognormal distributions.

The extra degree of freedom provided by the skewness of the initial distribution is of course beneficial and in particular the structures that are formed are well organised into filamentary structures. These simulations look highly promising.

So is the standard CDM model dead? It certainly has problems but source of the problems is easy to identify: the rather naive notion of biasing. It is conceivable that a better model of galaxy formation which automatically dealt with the biasing that has been put in to model the luminosity formation process could be made to work. The bias parameter was introduced to solve a problem: the amplitude of the velocities on the small scales were too great if the normalization was performed on the assumption that the mass and light distributions are identical ($b = 1$). Increasing the large scale amplitude in order to solve the problems that appear on large scales therefore has bad effects on the smaller scales. The small scale correlation function amplitude becomes too great and the velocity dispersion increases. That conclusion is however predicated on the assumption that CDM describes the small scale evolution correctly. Other effects, like dynamical friction and mergers in systems of galaxies that are not accurately modelled in the simulations could be taking place. So we must await even bigger ($N > 10^6$ or 10^7 -body models).

4.2.2 CDM with "gas"

There have been a number of attempts to take CDM further and consider gas flows in the potentials created by the dark matter (Carlberg and Couchman, 1989). Three dimensional simulations lack resolution and so Klypin et al. (1990) have done a high resolution CDM simulation in 2 dimensions, using a cloud-in-cell method to solve the equations of motion of the dark matter. The baryonic component is supposed to be glued to the dark matter, and its thermal history is computed along particle trajectories. In a 50 Mpc. box, the resolution is 50-100 kpc. That is the advantage of working in two dimensions.

The simulations show remarkable large scale filaments and voids, reminiscent of the de Lapparent et al. (1986) picture. This conclusion is supported by the adhesion model simulations of CDM (Weinberg and Gunn, 1990).

The Klypin et al. simulations show a remarkable feature: the overall structure of the model is not dominated by the smallest scales in the simulation. The galaxies lie along large scale filamentary structures (though because of the two dimensional nature of the simulation, we cannot say whether these features would be filament like or sheet like in three dimensions). This effect is apparently due to the effect of the velocity field correlations (Klypin, private communication). It will be interesting to see if this conclusion is borne out by very large three dimensional simulations.

4.2.3 Adhesion Model

The "pancake" theories for galaxy formation are well described by an elegant analytic approximation to the evolution of cosmic structure first proposed by Zeldovich (1970). In that approximation the position x_i of a particle in *comoving coordinates* is given relative to its position q_i at some starting time t_0 by the expression:

$$x_i = q_i - \beta(t) \left. \frac{\partial S}{\partial q_j} \right|_{t_0}, \tag{62}$$

where $S(q_i, t_0)$ is the velocity potential field at the time t_0. The form of the function $\beta(t)$ depends on the cosmological model, but in the case $\Omega = 1$ it is simply $\beta(t) = (t/t_0)^{2/3}$.

The peculiar velocity of a particle initially at point q is given in terms of S by

$$\mathbf{V} = a(t)\dot{\beta}(t)\frac{\partial S_0(\mathbf{q})}{\partial \mathbf{q}}, \tag{63}$$

and S is directly related to the density fluctuation amplitude at time t_0:

$$\frac{\partial \rho}{\rho} = -\beta \frac{\partial^2 S_0}{\partial \mathbf{q}^2}. \tag{64}$$

From the equation we see that the approximation is essentially a ballistic approximation in comoving coordinates with respect to the cosmic β-time. The gravitational effects of the surrounding mass distribution is not taken into account except insofar as gravity was responsible for causing the conditions at t_0. The approximation agrees with linear theory for the growth of small amplitude density contrasts. An improvement on this simple form of the approximation has been given by Buchert (1989).

The major problem with the Zeldovich solution is the fact that when the particle orbits intersect, a shock wave should form, dissipating the kinetic energy of the colliding streams (and forming the "pancakes" which fragment to make the galaxies). The dissipation is not a part of the approximation and so the streams penetrate and the pancakes get thicker after a time. Gurbatov et al. (1985, 1989) found a way of including dissipation in the Zeldovich approximation and at the same time reducing it to a set of equations well known in the one dimensional case as "Burgers" equation. If we write the Zeldovich approximation as a fluid flow, then the equation of motion is

$$\frac{\partial v_i}{\partial \beta} + v_j \frac{\partial v_i}{\partial x_j} = \nu \nabla^2 v_i. \tag{65}$$

The $\nu\nabla^2\mathbf{v_i}$ "viscosity" term is introduced to prevent orbit crossing. Note that the 'time' is β-time.

Several things should be noted about this equation. Firstly, there are no forces on the right hand side due to either pressure or gravity. This reflects the way in which the Zeldovich approximation is a ballistic approximation. Secondly, there is no explicit appearance of the density as would have been expected if ν were a real viscosity. This has an important consequence: the equation conserves velocity rather than momentum in the comoving system. This may cause systematic deviations between the adhesion approximation and N-body simulations that start from the same initial conditions. Thirdly, the equation has an analytic solution. If an explicit density dependence were introduced, there would be no analytic solution and the method would have little to commend it.

The equation has an analytic solution (given in terms of a rather uninformative Green's function). Numerical simulations in two dimensions using the Burgers approximation are relatively straightforward to implement, but there are some problems in three dimensions (Nusser and Dekel, 1990). An alternative scheme avoiding such problems (and directly incorporating biasing) have been developed by Appel and Jones (in preparation).

The "adhesion model" avoids the crossing streams problem and provides the possibility of making analytic studies of cosmological models with arbitrary spectra of inhomogeneities. The approximation is still limited in that it is purely kinematic, the force of gravity plays no part in the evolution of the density field. It is also formally valid only as long as the density contrast is not too high. This last problem was handled by Kofman, Pogosyan and Shandarin (1989) by using a second order modification of the Zeldovich approximation (there is great scope for work in that direction). So the major problem remains the fact that the orbits of the particles are not gravitationally deflected. This has consequences as can be seen in the comparisons (Kofman et al., 1989; Weinberg and Gunn, 1990) that have been made between the adhesion approximation and N-body calculations: the structural features are all there, but they are frequently in the wrong place.

The adhesion approximation has been used by Weinberg and Gunn (1990) to simulate galaxy redshift surveys to a magnitude limit of 15.5, starting from a CDM spectrum and using a bias parameter $b = 2$. The results show remarkable large scale structures. (Though, because of the way the adhesion approximation works, there are no "fingers of God" in the pictures). Park's (1990) very large N- body simulations with the same bias parameter confirm this. Park does express a preference for low Ω_0 on the basis of the appearance of the simulation.

4.2.4 Classical Pancakes

The pancake theory is the archetype theory in which the galaxies form after the large scale structure has been created. The theory had the merit that it was relatively straightforward to do simple numerical simulations based on the Zeldovich's approximation to the evolution of small amplitude density perturbations. Pictures of large scale structure could easily be evolved, and, as it turned out five years later, these bore a striking resemblance to the pictures published by de Lapparent et al. (1986). The basic review of the pancake theory is that of Shandarin and Zeldovich (1989).

The pancake theory is based on a spectrum of primordial density fluctuations that has no high frequencies. The spectral cutoff is supposed to be on scales larger than clusters of galaxies, and so the first thing to form is the large scale structure. In a purely baryonic cosmology with $\Omega_0 = 0.1 - 0.2$ there is a natural cutoff on such scales provided by the damping mechanisms that operate prior to the recombination epoch. However, it turns out that such large amplitudes are required (because Ω_0 is low) that the theory violates the limits on the isotropy of the microwave background radiation.

The amplitude problem might be solved by adding dark matter, but it must be a light particle in order that the cutoff scale be large. For a period of time there was a degree of enthusiasm about the possibility the neutrino had a mass of several eV's, and that would have served well to achieve $\Omega_0 = 1$ while at the same time giving a large characteristic mass. Simulations of the neutrino based theory (Klypin and Shandarin, 1983; Centrella and Melott, 1984; Centrella et al., 1988) gave clear indications of how the large scale structure might arise.

There are several reasons for the loss of general support for this theory, not the least of which is the fact that the most likely dark matter candidate needed by the theory has been almost, but not quite, ruled out (Scherrer, Melott and Bertschinger, 1989). There may still be problems with the microwavebackground, and there may be serious problems with the fact that galaxies should generally form rather late in this theory.

4.2.5 Decaying WIMPS

One method of saving the pancake theory may lie in supposing that $\Omega_0 = 1$ is made up of decaying weakly interacting massive particles ('WIMPS'). Doroshkevich has been an advocate of model universes pervaded by *unstable* dark matter (a heavy neutrino), so that $\Omega_{tot} = 1$ and $\Omega_B = 0.1$. Numerical simulations by Doroshkevich, Klypin, and Khlopov (1988, 1989) show that galaxies still form in the shocked pancakes, but at much earlier times than in the standard heavy-neutrino theory. (The epoch of formation depends on lifetime of decaying particle).

Decay of matter slows the growth of perturbations, but the decay occurs just before start of nonlinear stage of perturbation growth. (Otherwise the theory encounters a number of difficulties (Efstathiou, 1985; Flores et al., 1986; Vittorio and Silk, 1985)). The amplitudes required appear to cause no problems for the microwave background anisotropy limits.

4.3 Formation of Galaxy Clusters

Kaiser (1986) considers that clusters formed quite recently and that material beyond $\sim 1.5h^{-1} - 2h^{-1}$ Mpc, may still be infalling. A test of this might be the detection of subclustering in the outer regions of rich galaxy clusters (West and Bothun, 1990).

The idea that galaxy clusters, and in particular their outer regions, may contain information about their past is an important one. West et al., (1988, 1989) have examined systematic properties of simulated clusters, in the hope of recognizing different initial conditions. They have covered a large part of the cosmological parameter space. They have looked, for example, at $n = -2 - 1, 0$ and $n = 0$ pancake scenarios and they even do a universe with $\Omega_{baryon} = \Omega_{tot} = 0.15$ and $n = 0$. The novelty of the approach was to use low resolution simulations to locate clusters, followed by high resolution simulations to discover their properties. (An alternative approach would be to use Bertschinger's (1987) method of setting up initial conditions).

They look at density and velocity dispersion profiles, subclustering and cluster alignment. Perhaps unsurprisingly, they find that the central regions of galaxy clusters yield little information about initial conditions. The strongest tests of clustering theories lie in observing alignments of the clusters with each other and their surroundings, and in the amount of subclustering present in the outer regions. Simulations with dark matter show a rapid segregation of light and dark matter, which causes a systemic change of derived mass to light ratio with appealing to any bias mechanism. The subject is well reviewed by West (1989).

The role of dynamical friction during the cluster formation process was considered by Carlberg, Couchman and Thomas (1990) and by Carlberg and Duninski (1991). They argue that the effect of dynamical friction is to lower the velocity dispersion of the galaxies in a cluster relative to the velocity dispersion of the collisionless dark matter. The ratio of the velocity dispersions is called by them the velocity bias. The reduced velocity dispersion gives rise to a steeper light profile in clusters.

Evrard (1989) argues that the CDM theory has a problem in generating clusters of galaxies having velocity dispersion in excess of ~ 1000 km s^{-1}, of which there are several examples. Peebles, Daly and Juskiewicz (1989) amplify this. These claims are countered by Frenk et al. (1990) saying that there is an observational problem in knowing what the distribution of velocity dispersions of rich clusters of galaxies really is. There is even the suggestion that estimates of the velocity dispersions of such distant galaxy clusters may be hampered by projection effects.

There has been a lot of work done on the origin of the morphological types of galaxies in different environments and how these are modified during the evolution of the cluster via mergers, gas stripping and other processes. (See, for one random example, Evrard, Silk and Szalay (1990)).

4.4 Understanding Large Scale Structure

The numerical simulations certainly show how the large scale structures form, and they allow us to test different hypotheses regarding the initial spectrum, biasing, cosmological constants and whatever other parameters come into describing the universe. They do not however *explain* why the structure is the way it is. In other words, why do we see voids bounded by sheet-like structures? Until recently, the models were limited to several tens of thousands of particles, or at most 250,000. As impressive as this number seems, it still imposes a major limitation on our ability to resolve the structures on scales as small as galaxies while still looking at the largest structures. There is a lot of room for trying to develop a mathematical understanding of the process of structure formation.

The pioneering paper in this area was undoubtedly the work of Press and Schechter (1974, "PS") who through a very simple argument were able to calculate the mass spectrum of objects that form from a given spectrum of density fluctuations at the epoch of recombination. They argued that structure formation is a hierarchical process, the levels of which are determined by thresholding the density fluctuation field. The process stops when a limiting threshold is reached. At first, the limiting threshold was fixed by timescale arguments, but that idea was later changed when biasing was introduced as the mechanism whereby galaxies and clusters were discriminated as luminous objects (Kaiser, 1984; Bardeen, Bond, Kaiser and Szalay, 1986; see also Dekel and Rees, 1987).

The PS method is purely geometrical and is based only on knowing the fraction of the mass $f_\nu(M)$ in the universe at recombination that has density contrast in excess of ν standard deviations, σ_M, when the density field has been filtered with a window encompassing a mass

M. (Note that σ_M depends on mass). For Gaussian fluctuations in density:

$$f_\nu(M) = \frac{1}{2}\mathrm{erfc}\left[\frac{\nu(M)}{\sqrt{2}}\right]. \tag{66}$$

Note that in general ν will depend on the mass scale, as would happen in the case of constant $\delta\rho/\rho$ thresholding. Biased galaxy formation in its simple form only needs constant ν thresholding.

The number of objects in the mass interval M to $M + dM$ is, according to the PS ansatz:

$$\begin{aligned} N(M)dM &= 2\frac{\rho}{M}[f_\nu(M) - f_\nu(M+dM)]dM \\ &= 2\frac{\rho}{M}\frac{df_\nu}{dM}dM - \frac{1}{M}e^{-\nu^2/2}\frac{d\nu}{dM}dM. \end{aligned} \tag{67}$$

The first of these equations says that the numer of objects appearing in a given mass interval is simply the number of above threshold regions that appear when the window radius is changed. The factor 2 is a famous "fudge factor" included to account for infalling material. The second equation is merely a rewrite of the first, showing explicitly how the mass dependence of the threshold ν comes in.

The philosophy behind these equations is however flawed, as can be seen from numerical simulations of what the PS procedure is actually counting (Appel and Jones, 1990). Decreasing the window radius, for example, does not necessarily lead to the birth of objects whose mean size is known, most regions simply shrink in size as the window radius is increased.

The Press-Schechter method takes no account of the way in which gravity works on the mass distribution, or the way mass is converted into stars. Nevertheless, Efstathiou and Rees (1988) showed that, despite the naivety of the approach, the Press-Schechter formalism provided an acceptable fitting function to the mass function of objects identified in N-body experiments on the basis of a "friend-of-friends" algorithm. It should be noted that this way of identifying bound objects in N-body simulations is not in fact what is calculated by the PS threshold-based argument. In other words, the PS approach provides a pretty good fitting formula. (See also Carlberg and Couchman (1989)).

The subject has a long history (Couchman, 1987a,b; Martínez-González and Sanz, 1988; Lucchin and Matarrese, 1988; Appel and

Jones, 1990; Peacock and Heavens, 1990; to name but a few). Appel and Jones (1990) presented a new definition for what an incipient galaxy should be when looking at a random density field. In their definition, the window radius should adapt itself as a function of position: the window radius is increased until the local density maximum disappears below some chosen threshold. That radius then defines (up to a constant of proportionality) the mass of the object. By construction when using this prescription, objects of small size are rarely embedded inside bigger objects, thus solving the "nesting problem".

The "nesting problem" is solved by Peacock and Heavens (1990) in an entirely different way, though like most other authors they accept the basic tenet that the number distribution is given by the change in the occupied volume $f_\nu(M)$ with M.

A recent paper by Bond, Cole, Efstathiou and Kaiser (1991) looks at the way peaks in the density field appear and disappear as the window function radius is changed. In this sense the approach is rather like that of Appel and Jones (1990), but their treatment does not make the simplifying assumption that an object is defined by the disappearance of a peak when the window radius is increased. What is interesting is that, in their simplest model, they recover precisely the Press-Schechter formula!

5 CONCLUSIONS

It is difficult to 'conclude' about what is an ongoing and ever growing part of physics. What I hope is apparent from these lectures is that the interaction between theory and observation is very strong. The data being acquired today is constraining the theories we have developed.

Of course that could be because our theories are somewhat naive, but all that is required is to be willing to update our ideas as new data makes its impact. The key question today seems to be whether the Cold Dark Matter theory will survive after playing such a prominent role over the past years. I think that people are too eager to pronounce it dead. There are very many places where the theory is rather weak (biasing must be the main weakness), and there are equally many places where things can be changed without dramatically altering the whole concept (I think of introducing non-Gaussian perturbations, for example). What bothers me most is that we have no strong, alternative to take the place of CDM. Even if we were to go back to Peebles' open baryon-dominated universe with initial isocurvature perturbations having an $n = -1$ power spectrum, we would still have to fix it in order to get around the microwave background anisotropy constraints.

I think the future here lies in getting bigger and better N-body models with an attempt to inject more 'realism' into the galaxy formation process. It might be that CDM is fine if we do that, except that we are normalizing everything to the large scales.

From the point of view of the surveys — we obviously need more of them. It is impossible to assess the Broadhurst et al. pencil beam surveys without a better basis for doing statistics. My present inclination is to take that data at face value and see whether the N-body experiments can produce such an effect. I would like the comparison to be done statistically, not an eyeball test claiming that two pictures look pretty much the same.

Do Great Attractors exist and if so do they pose a problem for the standard CDM picture? That is another poorly phrased question, for there is certainly something out there perturbing the Hubble flow. If we accept the data at face value, we will have to make the amplitude (if not the spectrum) of the primordial fluctuations fit the large scales. That is where the models are best understood and where the data is probably clearest. As pointed out by numerous authors, that leaves us with a problem on the small scales, but I am prepared to say that we do not really understand what is going on there. We certainly do not understand what is involved in the formation of clusters (it has been suggested that dynamical friction and mergers will play a role there), and we understand even less of what is happening on galaxy scales. The present models are too simple.

So cosmology is not, in my opinion, in a state of crisis — at least not yet! It is in a state of rapid development and that is what makes it exciting from both the observational and theoretical point of view.

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