The K correction

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ABSTRACT

The K correction "corrects" for the fact that sources observed at different redshifts are, in general, compared with standards or each other at different rest-frame wavelengths. It is part of the relation between the emitted- or rest-frame absolute magnitude of a source in one broad photometric bandpass to the observed-frame apparent magnitude of the same source in another broad bandpass. This short pedagogical paper provides definitions of and equations for the K correction.

1. Introduction

The expansion of the Universe provides astronomers with the benefit that recession velocities can be translated into radial distances. It also presents the challenge that sources observed at different redshifts are sampled, by any particular instrument, at different rest-frame frequencies. The transformations between observed and rest-frame broad-band photometric measurements involve terms known as "K corrections" (Humason, Mayall, & Sandage 1956; Oke & Sandage 1968).

Here we define the K correction and give equations for its calculation, with the goals of explanation, clarification, and standardization of terminology.

In what follows, we consider a source observed at redshift z, meaning that a photon observed to have frequency ν_o was emitted by the source at frequency ν_e with

$$\nu_e = [1+z]\,\nu_o \quad . \tag{1}$$

The apparent flux of the source is imagined to be measured through a finite observed-frame bandpass R and the intrinsic luminosity is imagined to be measured through a finite emitted-frame bandpass Q. The K correction is used in relating these two quantites.

Technically, the K correction described here includes a slight generalization from the original conception: The observed and emitted-frame bandpasses are permitted to have arbitrarily different shapes and positions in frequency space (as they are in, *e.g.*, Kim et al 1996). In addition, the equations below permit the different bandpasses to be calibrated to different standard sources.

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2. Equations

Consider a source observed to have apparent magnitude m_R when observed through photometric bandpass R, for which one wishes to know its absolute magnitude M_Q in emitted-frame bandpass Q. The K correction K_{QR} for this source is *defined* by

$$m_R = M_Q + DM + K_{QR} \quad , \tag{2}$$

where DM is the distance modulus, defined by

$$DM = 5 \log_{10} \left[\frac{D_L}{10 \text{ pc}} \right] \quad , \tag{3}$$

where D_L is the luminosity distance (*e.g.*, Hogg 1999) and 1 pc $= 3.086 \times 10^{16}$ m.

The apparent magnitude m_R of the source is related to its spectral density of flux $f_\nu(\nu)$ (energy per unit time per unit area per unit frequency) by

$$m_R = -2.5 \log_{10} \left[\frac{\displaystyle\int \frac{\mathrm{d}\nu_o}{\nu_o} f_\nu(\nu_o)\, R(\nu_o)}{\displaystyle\int \frac{\mathrm{d}\nu_o}{\nu_o} g_\nu^R(\nu_o)\, R(\nu_o)} \right] \quad , \tag{4}$$

where the integrals are over the observed frequencies ν_o; $g_\nu^R(\nu)$ is the spectral density of flux for the zero-magnitude or "standard" source, which, for Vega-relative magnitudes, is Vega (or perhaps a weighted sum of a certain set of A0 stars), and, for AB magnitudes (Oke & Gunn 1983), is a hypothetical constant source with $g_\nu^{\mathrm{AB}}(\nu) = 3631$ Jy (where 1 Jy $= 10^{-26}$ W m^{-2} Hz^{-1} $= 10^{-23}$ erg cm^{-2} s^{-1} Hz^{-1}) at all frequencies ν; and $R(\nu)$ describes the bandpass, as follows:

The value of $R(\nu)$ at each freqency ν is the mean contribution of a photon of frequency ν to the output signal from the detector. If the detector is a photon counter, like a CCD, then $R(\nu)$ is just the probability that a photon of frequency ν_o gets counted. If the detector is a bolometer or calorimeter, then $R(\nu)$ is the energy deposition $h\,\nu$ per photon times the fraction of photons of energy ν that get absorbed into the detector. If $R(\nu)$ has been properly computed, there is no need to write different integrals for photon counters and bolometers. Note that there is an implicit assumption here that detector nonlinearities have been corrected.

The absolute magnitude M_Q is defined to be the apparent magnitude that the source *would* have if it were 10 pc away, at rest (*i.e.*, not redshifted), and compact. It is related to the spectral density of the luminosity $L_\nu(\nu)$ (energy per unit time per unit frequency) of the source by

$$M_Q = -2.5 \log_{10} \left[\frac{\displaystyle\int \frac{\mathrm{d}\nu_e}{\nu_e} \frac{L_\nu(\nu_e)}{4\pi\,(10 \text{ pc})^2}\, Q(\nu_e)}{\displaystyle\int \frac{\mathrm{d}\nu_e}{\nu_e} g_\nu^Q(\nu_e)\, Q(\nu_e)} \right] \quad , \tag{5}$$

where the integrals are over emitted (*i.e.*, rest-frame) frequencies ν_e, D_L is the luminosity distance, and $Q(\nu)$ is the equivalent of $R(\nu)$ but for the bandpass Q. As mentioned above, this does not

require $Q = R$, so this will lead to, technically, a generalization of the K correction. In addition, since the Q and R bands can be zero-pointed to different standard sources (*e.g.*, if R is Vega-relative and Q is AB), it is not necessary that $g_\nu^Q = g_\nu^R$.

If the source is at redshift z, then its luminosity is related to its flux by

$$L_\nu(\nu_e) = \frac{4\pi\, D_L^2}{1+z}\, f_\nu(\nu_o) \quad , \tag{6}$$

$$\nu_e = [1+z]\, \nu_o \quad . \tag{7}$$

The factor of $(1+z)$ in the luminosity expression (6) accounts for the fact that the flux and luminosity are not bolometric but densities per unit freqency. The factor would appear in the numerator if the expression related flux and luminosity densities per unit wavelength.

Equation (2) holds if the K correction K_{QR} is

$$K_{QR} = -2.5\, \log_{10} \left[[1+z] \frac{\displaystyle\int \frac{\mathrm{d}\nu_o}{\nu_o}\, f_\nu(\nu_o)\, R(\nu_o) \int \frac{\mathrm{d}\nu_e}{\nu_e}\, g_\nu^Q(\nu_e)\, Q(\nu_e)}{\displaystyle\int \frac{\mathrm{d}\nu_o}{\nu_o}\, g_\nu^R(\nu_o)\, R(\nu_o) \int \frac{\mathrm{d}\nu_e}{\nu_e}\, f_\nu\left(\frac{\nu_e}{1+z}\right)\, Q(\nu_e)} \right] \quad . \tag{8}$$

Equation (8) can be taken to be an operational definition, therefore, of the K correction, from observations through bandpass R of a source whose absolute magnitude M_Q through bandpass Q is desired. Note that if the R and Q have different zero-point definitions, the $g_\nu^R(\nu_e)$ in the numerator will be a different function from the $g_\nu^Q(\nu_o)$ in the denominator.

In equation (8), the K correction was defined in terms of the apparent flux $f_\nu(\nu)$ in the observed frame. This is the direct observable. Most past discussions of the K correction (*e.g.*, Oke & Sandage 1968; Kim et al 1996) write equations for the K correction in terms of either the flux or luminosity in the emitted frame. Transformation from observed-frame flux $f_\nu(\nu_o)$ to emitted-frame luminosity $L_\nu(\nu_e)$ gives

$$K_{QR} = -2.5\, \log_{10} \left[[1+z] \frac{\displaystyle\int \frac{\mathrm{d}\nu_o}{\nu_o}\, L_\nu([1+z]\nu_o)\, R(\nu_o) \int \frac{\mathrm{d}\nu_e}{\nu_e}\, g_\nu^Q(\nu_e)\, Q(\nu_e)}{\displaystyle\int \frac{\mathrm{d}\nu_o}{\nu_o}\, g_\nu^R(\nu_o)\, R(\nu_o) \int \frac{\mathrm{d}\nu_e}{\nu_e}\, L_\nu(\nu_e)\, Q(\nu_e)} \right] \quad . \tag{9}$$

In the above, all calculations were performed in frequency units. In wavelength units, the spectral density of flux $f_\nu(\nu)$ per unit frequency is replaced with the spectral density of flux $f_\lambda(\lambda)$ per unit wavelength using

$$\nu\, f_\nu(\nu) = \lambda\, f_\lambda(\lambda) \quad , \tag{10}$$

$$\lambda\, \nu = c \quad , \tag{11}$$

where c is the speed of light. The K correction becomes

$$K_{QR} = -2.5\,\log_{10}\left[\frac{1}{[1+z]}\,\frac{\int \mathrm{d}\lambda_o\,\lambda_o\,f_\lambda(\lambda_o)\,R(\lambda_o)\int \mathrm{d}\lambda_e\,\lambda_e\,g_\lambda^Q(\lambda_e)\,Q(\lambda_e)}{\int \mathrm{d}\lambda_o\,\lambda_o\,g_\lambda^R(\lambda_o)\,R(\lambda_o)\int \mathrm{d}\lambda_e\,\lambda_e\,f_\lambda([1+z]\lambda_e)\,Q(\lambda_e)}\right] \quad , \tag{12}$$

where, again, $R(\lambda)$ is defined to be the mean contribution to the detector signal in the R bandpass for a photon of wavelength λ and $Q(\lambda)$ is defined similarly. Note that the hypothetical standard source for the AB magnitude system, with $g_\nu^{\mathrm{AB}}(\nu)$ constant, has $g_\lambda^{\mathrm{AB}}(\lambda)$ not constant but rather $g_\lambda^{\mathrm{AB}}(\lambda) = c\,\lambda^{-2}\,g_\nu^{\mathrm{AB}}(\nu)$.

Again, transformation from observed-frame flux $f_\lambda(\lambda_o)$ to emitted-frame luminosity $L_\lambda(\lambda_e)$ gives

$$K_{QR} = -2.5\,\log_{10}\left[\frac{1}{[1+z]}\,\frac{\int \mathrm{d}\lambda_o\,\lambda_o\,L_\lambda\left(\frac{\lambda_o}{1+z}\right)\,R(\lambda_o)\int \mathrm{d}\lambda_e\,\lambda_e\,g_\lambda^Q(\lambda_e)\,Q(\lambda_e)}{\int \mathrm{d}\lambda_o\,\lambda_o\,g_\lambda^R(\lambda_o)\,R(\lambda_o)\int \mathrm{d}\lambda_e\,\lambda_e\,L_\lambda(\lambda_e)\,Q(\lambda_e)}\right] \quad . \tag{13}$$

Equation (13) becomes identical to the equation for K in Oke & Sandage (1968) if it is assumed that $Q = R$, that $g_\nu^Q = g_\nu^R$, that the variables λ_0, $F(\lambda)$, and $S_i(\lambda)$ in Oke & Sandage (1968) are set to

$$\begin{aligned} \lambda_0 &= \lambda_e \quad , \\ F(\lambda) &= L_\lambda(\lambda) \quad , \\ S_i(\lambda) &= \lambda\,R(\lambda) \quad , \end{aligned} \tag{14}$$

and that the integrand λ is used differently in each of the two integrals. Similar transformations make the equations here consistent with those of Kim et al (1996), although they distinguish between the classical K correction and one computed for photon counting devices (an unnecessary distinction); their most similar equation is that given for K_{xy}^{counts}.

3. Discussion

To compute an accurate K correction, one needs an accurate description of the source flux density $f_\nu(\nu)$, the standard-source flux densities $g_\nu^R(\nu)$ and $g_\nu^Q(\nu)$, and the bandpass functions $R(\nu)$ and $Q(\nu)$. In most real astronomical situations, none of these is known to better than a few percent, often much worse. Sometimes, use of the AB system seems reassuring (relative to, say, a Vega-relative system) because $g_\nu^{\mathrm{AB}}(\nu)$ is known (*i.e.*, defined), but this is a false sense: In fact the standard stars have been put on the AB system to the best available accuracy. This involves absolute spectrophotometry of at least some standard stars, but this absolute flux information is

rarely known to better than a few percent. The expected deviations of the magnitudes given to the standard stars from a true AB system are equivalent to uncertainties in $g_\nu^{\rm AB}(\nu)$.

The classical K correction has $Q(\nu) = R(\nu)$ and $g_\nu^Q(\nu) = g_\nu^R(\nu)$. This eliminates the integrals over the standard-source flux density $g_\nu^R(\nu)$. However, it requires good knowledge of the source flux density $f_\nu(\nu)$ if the redshift is significant. Many modern surveys try to get $R(\nu) \sim Q([1+z]\nu)$ so as to weaken dependence on $f_\nu(\nu)$, which can be complicated or unknown. This requires good knowledge of the absolute flux densities of the standard sources if the redshift is significant. This kind of absolute calibration is often uncertain at the few-percent level or worse.

Note that if equation (2) is taken to be the definition of the K correction, then the statement by Oke & Sandage (1968) that the K correction "would disappear if intensity measurements of redshifted galaxies were made with a detector whose spectral acceptance band was shifted by $1+z$ at all wavelengths" becomes incorrect; the correct statement is that the K correction would not depend on the source's spectrum $f_\nu(\nu)$.

It is a pleasure to thank Bev Oke for useful discussions. This research made use of the NASA Astrophysics Data System, and was partially supported by funding from NASA and NSF.

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